Mathematics reveals patterns, order and relationships which help us to understand the world around us. It is an increasingly varied discipline that deals with number, spatial sense, data, measurement, and observations requiring inference, deduction, modeling and communication. Mathematics is of fundamental importance in the workplace and in personal life contexts. Today, mathematical concepts are rapidly expanding into many other subject disciplines. Effective classrooms are places of inquiry where mathematics is fostered, where teachers are inspiring, knowledgeable in mathematics, and skilled in creating rich learning environments.

The Effective Mathematics Classroom Environment

- Both teachers and students demonstrate a positive attitude towards mathematics
- Is a safe, supportive and respectful classroom
- Includes engaging, challenging and authentic mathematics
- High expectations are held by teachers and students
- Provides collaborative learning opportunities
- Student mathematical thinking and learning is evident and posted
- Students have easy access and choice to learning tools, mathematics resources and technologies

Professional Learning In Mathematics

Teachers engage in collaborative professional learning, such as:

- Co-planning
- Co-teaching
- Lesson study
- Webinars
- Modular learning
- Self-directed Professional learning communities

Teachers study relevant topics such as:

- Engaging students in relevant challenging mathematics;
- Current pedagogy in mathematics e.g. three-part lesson, teaching through problem-solving, Bansho, Congress, Gallery Walk;
- Math content for teaching;
- The developmental continuum of math concepts and skills;
- The use of the technology in teaching and learning of mathematics;
- Effective assessment tools and strategies for mathematics teaching & learning.

The Learning Destinations For All Students in Mathematics

Successful planning and teaching enable students consistently and independently

- Understand, remember, and apply mathematical concepts
- Articulate clear understanding of mathematical concepts and ideas and give clear examples of use
- Apply concepts, skills, and strategies to propose solutions to problems
- Analyze problems, use a variety of strategies & processes to find solutions, and be able to check and evaluate effectiveness of processes used
- Work effectively alone and with others
- Communicate effectively using words, symbols, and representations
- Connect ideas to self, others and other ideas/tasks
- Use “mathematical habits of mind”, e.g. problem solving, reasoning, number sense, logical thinking.
- Demonstrate learning and thinking in a range of ways including:
  - Products e.g. work samples, tests, quizzes;
  - Observations e.g. class work, demonstrations, performance tasks, teacher observations;
  - Conversations e.g. discussion, written reflections, journal entries, conference
  - Interventions

(Taken from TDDB Mathematics Foundation Statement P.004 CUR)

Characteristics Of Mathematically Literate Students

- Can communicate their mathematical thinking and can understand the mathematical reasoning of others
- Have a sense of numbers and are sufficiently efficient in their work that their thinking guides as they progress towards a solution
- To make sense of mathematics
- Make connections between concepts and see pattern throughout mathematics
- Are willing to persevere in order to understand and solve mathematical problems
- Are developing depth and flexibility in their thinking

The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario
• Is focused on having students make sense of mathematics;
• Is based on problem solving and investigation of important mathematical concepts;
• Begins with the students’ understanding and knowledge of the topic;
• Includes students as active, rather than passive participants in their learning;
• Allows students to engage in a variety of problem solving experiences to develop conceptual and procedural understanding and skills through exploration, investigation, direct instruction and practice;
• Has students communicate and investigate their thinking through ongoing discussion;
• Includes all students, whether in the choice of problems or in the communicating of mathematical ideas;
• Incorporates ongoing assessment of student understanding to guide future instruction;
• Includes opportunities for students to apply math knowledge and skills in other subject areas;
• Engages students in investigations or questions prompted by literature or a topic in other subject areas;
• Uses optimal groupings to encourage collaborative discourse and critical thinking.

COMPREHENSIVE MATHEMATICS PROGRAM

INSTRUCTIONAL STRATEGIES IN MATHEMATICS

Gallery Walk
• Students and teachers circulate to examine student recorded solutions to a lesson problem
• Students read solutions and give oral and written feedback
• Teachers assess the range of mathematical thinking in the different solutions
• Teachers use the information to determine the focus of consolidation and next steps for planning

Math Congress
• Two or three student solutions are used to conduct a whole class discussion that will develop every student’s mathematical thinking
• Teachers use student solutions to prompt them to reason about big math ideas
• Specific ideas and strategies are generalized and connections made to previous math discussions and learning
• Students defend and support their solutions and thinking
• Teachers use questioning to prompt all students to reason and generalize based on the lesson goals

Bansho
• Means board writing in Japanese
• Math thinking is organized and recorded as it occurs in the lesson
• Mathematical expressions, diagrams, solutions and strategies are recorded for all to engage in
• Various solutions are organized and compared through teacher and student questioning
• Students deepen their mathematical thinking through comparing, generalizing and summarizing

Assessment For and As Learning

K-2
• Learning Goals
• Success Criteria
• Descriptive feedback
• Student interviews
• Observational data
• Student work (oral, written representations)
• Student models using manipulatives
• Exit Cards
• Reflective Journals
• Peer and self assessment items
• Portfolio
• Peer editing

3-5
• Learning Goals
• Success Criteria
• Descriptive feedback
• Student interviews
• Observational data
• Student work (oral, written)
• Student models using manipulatives
• Exit Cards
• Reflective Journals
• Peer and self assessment items
• Portfolio

6-9
• Learning Goals
• Success Criteria
• Descriptive feedback
• Student interviews
• Observational data
• Student work (oral, written)
• Student models using manipulatives
• Exit Cards
• Reflective Journals
• Peer and self assessment items
• Portfolio

10-12
• Learning Goals
• Success Criteria
• Descriptive feedback
• Student interviews
• Observational data
• Student work (oral, written)
• Student models using manipulatives
• Exit Cards
• Reflective Journals
• Peer and self assessment items
• Portfolio

Teacher Moderation
• An examination of student work with colleagues to compare interpretations of student results, and confirm judgments about levels of achievement

Peer and Self Assessment
• Assessment of student work or learning processes by self or classmates using the established success criteria

ASSESSMENT FOR AND AS LEARNING

COMPONENTS
• Instructions
• Descriptors
• Look fors

COMPONENTS
• Data collection methods

Teacher Moderation
• Instruction

Look fors
• Professional learning is focused on student work and taking student(s) to the next level of learning
• Structured regular opportunities to examine student work collaboratively in the classroom setting
• Calibrate standards of expectations and practice among teachers within and across grades
• Develop school-wide beliefs, values and practices that support students being well and successful; and
• School staff can explain what is the difference that makes the difference for their students

Peer and Self Assessment
• Instruction

Look fors
• Students have regular and structured opportunities to speak to each other about their progress and their work
• Students can describe what success looks like based on set criteria; and
• Students have a clear understanding and can articulate why they are learning, what and how they are learning
LEARNING SKILLS/MATHEMATICAL PROCESSES

- Are embedded within individual math strands;
- Students should be actively engaged in applying these process expectations throughout all math classes and courses;
- Processes are interconnected and integrated, particularly Problem Solving & Communicating;
- Are fundamental to students’ construction of their knowledge and skills related to mathematics; and
- Allow students to monitor and reflect on their learning (metacognition).

COMPONENTS

K - 1 - 3
4 - 6
7 - 10
11 - 12

Problem Solving

- Children begin to develop and apply problem-solving skills as they work with concrete materials and geometric shapes;
- Apply developing problem-solving strategies as they pose and solve problems and construct investigations, to help deepen their mathematical understanding;
- Develop, select, and apply a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- Develop, select, apply, and combine a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

Reasoning and Proving

- Children apply reasoning skills to make and investigate conjectures;
- Apply developing reasoning skills to make and investigate conjectures;
- Develop and apply reasoning skills to make and investigate conjectures and construct and defend arguments;
- Develop and apply reasoning skills to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments;

Reflecting

- Children demonstrate that they are reflecting on and monitoring their thinking to help clarify what they are thinking,
- Demonstrate that they are reflecting on and monitoring their thinking to help clarify what they are thinking,
- Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem;
- Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem;

Selecting Tools and Computational Strategies

- Select and use a variety of computational strategies and appropriate computational tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
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Connecting

- Children begin to make connections among mathematical concepts and to situations or phenomena drawn from everyday contexts;
- Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from everyday contexts;
- Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from everyday contexts;
- Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from everyday contexts;

Representing

- Children create basic representations of simple mathematical ideas, make connections among them, and apply them to solve problems;
- Create basic representations of simple mathematical ideas, make connections among them, and apply them to solve problems;
- Create a variety of representations of mathematical ideas, make connections among them, and apply them to solve problems;
- Create a variety of representations of mathematical ideas, make connections among them, and apply them to solve problems;

Communicating

- Children communicate mathematical thinking orally, visually, and in writing, using everyday language, an emerging mathematical vocabulary, and a variety of representations;
- Communicate mathematical thinking orally, visually, and in writing, using everyday language, an emerging mathematical vocabulary, and a variety of representations;
- Communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, including basic mathematical conventions;
- Communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, including basic mathematical conventions;

COMPONENTS

Descriptive

- Authentic activity, exercise, problem or challenge that requires students to show what they know and can do. This task will address all four categories of the achievement chart;
- Includes multiple overall curriculum expectations;
- Addresses all four categories of the achievement chart;

Look Fors

- Authentic activity, exercise, problem or challenge that requires students to show what they know and can do. This task will address all four categories of the achievement chart;
- Includes multiple overall curriculum expectations;
- Addresses all four categories of the achievement chart;

ASSESSMENT FOR AND AS LEARNING

- Through facilitated discussions with students and teacher moderation of student work, teachers assess student learning, plan and provide feedback, and adjust instruction.
- What students should be able to do by the end of a period of instruction (e.g. a lesson, series of lessons, or subtask).
- A description of successful attainment of the learning goals that shows what success “looks like.”
- Success criteria are used to guide descriptive feedback that informs the next level of teaching and learning.

Student Work

- Student work used to share strategies and solutions, consolidate learning, prompt mathematical communication.
- Student work samples guide professional learning and decision making.

Learning Goals

- Based on overall curriculum expectations and big ideas in mathematics;
- Stated in clear language.

Success Criteria

- Co-constructed between teachers and students;
- Observable in the classroom;
- Accessible to students for reference and scaffolds their learning;
- Describes what success looks like without showing students which solutions or strategies to use;

Rich Assessment Task

- Through facilitated discussions with students and teacher moderation of student work, teachers assess student learning, plan and provide feedback, and adjust instruction.
- What students should be able to do by the end of a period of instruction (e.g. a lesson, series of lessons, or subtask).
- A description of successful attainment of the learning goals that shows what success “looks like.”
- Success criteria are used to guide descriptive feedback that informs the next level of teaching and learning.

Precise and Differentiated Instruction

- Teaching and learning experiences that respond to the individual learning needs of students;
- Instruction is adjusted based on the observations from continuous assessment of student work;
- Explicit descriptive feedback is provided to students, enabling them to set appropriate learning goals and improve their achievement.

Components

- Ongoing assessment
- Demonstrates what students already know
- Informs instruction that is differentiated and personalized
- Monitors students progress
- Ongoing self and peer assessment
- Provides feedback from students to other students
- Informs students of their own learning styles/preferences
- Allows for individualized goal setting

CONTINUED...
An information and technology-based society requires individuals who are able to think critically about complex issues, people who can "analyze and think logically about new situations, devise unspecified solution procedures, and communicate their solution clearly and convincingly to others." (Baroody, 1998, p. 2-1).

To prepare students to function in such a society, teachers have a responsibility to promote in their classrooms the experience of problem-solving processes and the acquisition of problem-solving strategies, and to foster in students positive dispositions towards problem solving.

**Teacher Actions**
- Activate students’ prior knowledge;
- Mentally engage students in the problem-solving situation by posing a thought-provoking problem;
- Model tool and strategy selection as appropriate;
- Gather diagnostic and/or formative assessment data through observation and questioning;
- Discuss and clarify the task, including having students restate the problem in their own words and ask questions;
- Establish expectations; and
- Establish a positive mathematics classroom climate.

**Student Actions**
- Participate in discussions;
- Propose strategies;
- Question the teacher and their classmates; and
- Make connections to and reflect on prior learning.

**Teacher Actions**
- Facilitate whole class discussion and sharing by:
  - √ providing hints and suggestions;
  - √ encouraging testing of ideas;
  - √ suggesting extensions of generalization;
  - √ asking probing questions;
  - √ answering students’ questions to clarify mathematical misconceptions (but avoid providing a solution to the problem);
  - √ encourage students to represent their thinking;
- Observe and assess;
- Encourage students to clarify ideas and to pose questions to other students (math talk); and
- Make connections with literacy and learning for life.

**Student Actions**
- Justify and explain their thinking and understanding with clarity and precision;
- Compare a variety of concrete, pictorial, and numerical representations;
- Listen and contribute to reflections on alternative approaches, different solutions, as well as extensions and connections;
- Reflect on their learning; and
- Retell, rephrase, and/or expand on mathematical ideas.

**Teacher Actions**
- Facilitate whole class discussion and reflection by:
  - √ probing students to summarize the discussion and emphasizing key points or concepts, i.e., “pulling out the math”;
  - Record key mathematical concepts, vocabulary, algorithms, strategies in a list; and
  - Make the learning from the lesson explicit.

**Student Actions**
- Articulate main points and ideas; and
- Express mathematical ideas and make connections.

**Teacher Actions**
- Provide opportunities to practice
  - √ skills, problem solving, visualization, communication, metacognition;
  - Plan the type and amount of practice based on the outcome of the lesson; and
  - Differentiate the type and amount of practice based on student needs.

**Student Actions**
- Participate actively in whole group, small group, or independent settings; and
- Record solutions, thinking, representations, strategies.