GRADE 12 CALCULUS AND VECTORS
PRACTICE EXAM

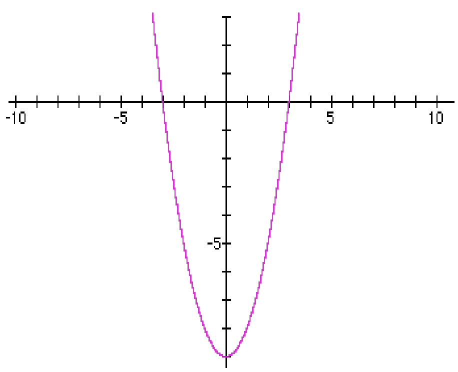
1. Evaluate the following limits.

a) $\lim\limits_{x \to 5} \dfrac{x-5}{x^2-25}$ b) $\lim\limits_{x \to \infty} \dfrac{-2x^2+7x-4}{5x^2+3x-2}$ c) $\lim\limits_{h \to 0} \dfrac{(3+h)^2+9}{h}$ d) $\lim\limits_{x \to 0} \dfrac{\sqrt{4+x}-2}{x}$

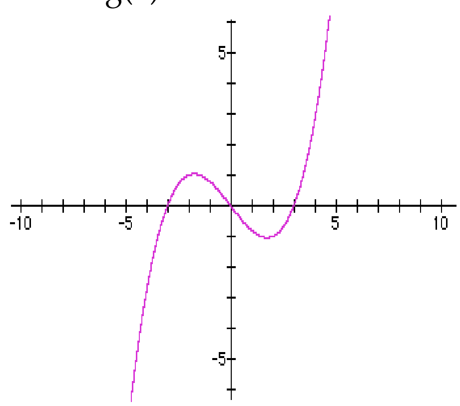
2. Find the equation of the tangent to the curve $f(x) = 2x^2 + 3x + 1$ at x = -4 from first principles.

3. Given the graphs of $f'(x)$ and g(x) below sketch the graphs of $f(x)$ and g′(x).

$f'(x)$



g(x)

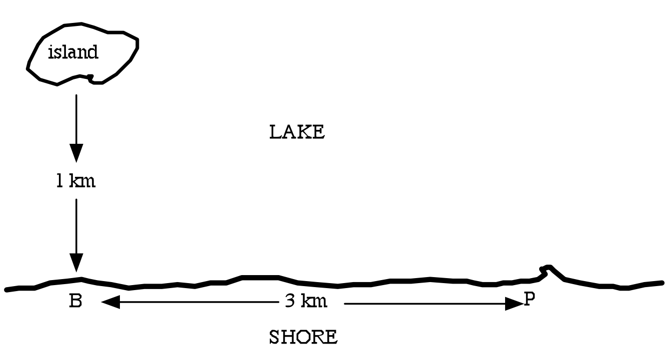


4. Find the derivatives of the following using the indicated methods.

a) $y = \dfrac{-8}{x^4}$, y' b) $y = \dfrac{(7x^{-3} - 6x + 1)}{(3x^3 - 2x)}$, $\dfrac{dy}{dx}$ c) $y = \left[\ln\left(\sin\left(\sqrt{4x^3}\right)\right)\right]^2$, $\dfrac{dy}{dx}$

d) $y = \dfrac{(3x^2+2)(e^{2x}-e)(\ln 3x + 4x)}{(\cos^3 x)}$, logarithmic differentiation

5. In a race to test strength, endurance and mathematical fitness, participants are required to carry a canoe from point P and at some point along the shore place the canoe in the water and paddle to an island that is 1 km offshore. A participant can paddle at 3km/h and run(while carrying the canoe) at 4 km/h. Determine the distance that the participant should run on land in order to reach the island in the shortest possible time.



6. A large vat in the shape of a right circular cone is being filled with molten aluminum at the rate of 20L per minute. Find the instantaneous rate of change of the height of the aluminum in the container at the moment the height is 50cm. ($1L = 1000cm^3$)

7. 200 N of force is applied on an object at an angle of 30 degrees to the direction of movement. If the object moves 6 meters determine the work done on the object using a type of vector multiplication.

8. A plane flies on a heading of $045°$ at a constant speed of 500 km/h. If the velocity of the wind 60 km/h on a bearing of $120°$, what is the velocity of the plane relative to the ground?

9. Sketch the sum of the following vectors in R^3 : $\vec{u} = [2,4,6]$ and $\vec{v} = [-5,7,3]$

10. Determine whether or not the following points are co-linear:
$J[2,6,2]$, $K[-1,3,0]$, $L[8,1,-2]$

11. The tails of vectors $\vec{a}$ and $\vec{b}$ form two sides of a parallelogram. The angle between the vectors is 30 degrees and the magnitudes are 5 and 10. Determine the area of the parallelogram.

12. Calculate the area of the parallelogram determined by the following vectors:
$\vec{a} = [1,4,3]$ and $\vec{b} = [3,1,-1]$

13. Determine the volume of the parallelepiped formed by the vectors $\vec{u} = [2,1,3]$, $\vec{v} = [1,-4,2]$ and $\vec{w} = [0,3,5]$

14. Use cross product to prove sine law.

15. Use dot product to prove cosine law.