Commentary

Teach Mathematics as it should be taught—as a creative art

Peter Taylor

Department of Mathematics and Statistics, Queen's University, peter.taylor@queensu.ca.
Correspondance: peter.taylor@queensu.ca; 613-533-2434

Abstract: The secondary-school mathematics curriculum is narrow in scope and technical in character; this is quite different from the nature of the discipline itself. As a result, it offers little inspiration to both students and teachers, and it provides students with a poor preparation for university mathematics courses. Stretching over the past century, and recently more than ever, there have been calls for change, for a curriculum that is true to the subject of mathematics as a creative art. While there are hopeful responses to this at the elementary level, there is almost nothing at the secondary level. Ironically it is felt that in order to prepare students for university calculus, the secondary curriculum simply has to be what it is. This is a myth that needs to be destroyed.

Keywords: secondary school; mathematics curriculum; creative arts; aesthetics; experience.

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1. Statement of the myth

I take it as understood that mathematics is an essential and even central discipline in the new curriculum. As a consequence, it is important that the teaching and learning of mathematics be held to a high standard. However it has frequently been remarked that in our K-12 school system, this high standard is not generally being met and that the overall level of mathematical knowledge and performance of our students is unacceptably low. In papers and articles written over the past 100 years, major authors have placed much of the responsibility for this on the nature of school mathematics, that it is narrow in scope and technical in character. The general response to this, either explicitly or more often implicitly, has been that school math has to be like this, that that is simply the nature of the discipline at the school level.

Mathematics is both a science and a creative art. I want to begin with a focus on its character as an art, and compare it with other creative arts such as painting, music, drama and English, perhaps most notably the last, as it has, in the school curriculum, the same central status as mathematics. These curricula are neither narrow nor particularly technical; they rest on the study and appreciation of what I can call "works of art," pieces that are discussed, analyzed and enjoyed, not only by scholars of the discipline (though they *are*), but by folks with a curious creative mind. They are indeed whole experiences that typically relate to our lives. They often carry a sophistication in both structure and meaning.

> There are other indications that math is different from all other subjects. When we ask students what math is, they will typically give descriptions that are very different from those given by experts in the field. Students will typically say it is a subject of calculations, procedures, or rules. But when we ask mathematician what math is, they will say it is the study of patterns that is an aesthetic, creative, and beautiful subject. Why are these descriptions so different? When we ask students of English literature what the

> subject is, they do not give descriptions that are markedly different from what professors of English literature would say [1] (pp. 21-22).

Now these works of art have analogues in mathematics—activities, problems and projects—that are of interest to mathematicians but that can also be enjoyed and wondered at by folks with a curious creative mind. But other than in exceptional circumstances, such mathematical works of art are not found in the school math curriculum. Quite simply, school mathematics is not what mathematicians do. I find it useful to state my myth in the context of this disjunction between mathematics and other creative arts.

The Myth. The school math curriculum is necessarily narrow in scope and technical in character. In this regard, it differs from curricula in the other creative arts which are based on the investigation, discussion and enjoyment of sophisticated works of arts. Analogous works exist in mathematics but these are not accessible to school students. The reasons for this are found in what is a fundamental difference in nature between mathematics and the other creative arts.

2. And what is this "fundamental difference"?—Automaticity.

What is it about mathematics that sets it apart? It is the idea that you can't do serious mathematics without the mastery of a body of technical skills.

> … Now there is just enough truth in this answer to have made it live through the ages. But for all its half-truth, it embodies a radical error which bids fair to stifle the genius of the modern world… The mind is never passive; it is a perpetual activity, delicate, receptive, responsive to stimulus. You cannot postpone its life until you have sharpened it [2] (p. 6).

In place of *mastery* I prefer the term *automaticity* [3], in part because it fits well with other common activities such as skating, riding a bicycle and playing a guitar. These all emphasize the principle that the working brain has limited capacity and can't successfully grapple with a sophisticated structure until an automatic component of the analysis can be given over to a non-analytical part of the brain. The point is that without an automatic facility with the required basic technical and even conceptual background, one simply cannot make any progress in a mathematical inquiry. That kind of automaticity takes time and practice—there is no "royal road."

Although this principle of technical automaticity also applies in the creative arts, it appears, for the most part, to be less critical there. For example, folks hold forth in the daily media on significant issues without much technical understanding of the main arguments or the texts on which the discussion is based—and in so doing they can nevertheless make a contribution. Amateur artists and dancers can give wonderful performances. But in mathematics, if you do not understand the notation and the basic results, it is almost impossible to make progress.

An important point to be made is that this automaticity needs to be practiced *in context*. In basketball, you can work on the jump shot again and again, but in the heat of the game everything changes. The drill has to be accompanied by the game.

However, the significant take-away is that in mathematics there are not really as many technical procedures that need to be automatic as one might think. I will talk here about a creative redesign of the curriculum that can reduce the technical components to the set that is needed for each work of art, leaving ample room for investigation and discovery. One needs to be sure that the discoveries are worthwhile, that the works of art are full of life and that ultimately (by the end of the K-12 sequence) the essential skills are in place. Much of the effort that was squandered in the infamous "math-wars," would have been better spent on such a curriculum redesign.

3. Why is the myth harmful?

What are the objectives of the mathematics curriculum, indeed what is the point of attending school? Such questions have, of course, been much discussed, but let's take the objective to be simply a full and rich life (and I will say more about this later).

> What this means is that at its best, schooling can be about how to make a life, which is quite different from how to make a living [4] (preface p. *x*).

Most students have a fairly narrow vision of the scope and potential of their lives so that the first task of the teacher must be to give them experiences that will widen this scope and develop their conceptual and technical powers.

For the great majority of our students the current school math curriculum fails to connect with their inner lives, with their "real" being. Even though many will put that alienation aside and "learn" the material (driven perhaps by the need to get good marks and ultimately good jobs) there is an underlying sense of disconnect and boredom. As a consequence, the material is not played with, it is not handled in a serious and playful manner, it is not learned in a way that will be available to the student, now or in the years ahead. This narrow technical interpretation of mathematics does a grave disservice to the expectant student, to the responsible teacher and to the wonderful subject of mathematics itself.

4. Rescuing the math curriculum—discovery learning.

The work of a mathematician might well be described as seeking to understand the structure of a sophisticated configuration of objects and relationships. In fact, in more concrete situations, this is a reasonable description of what all of us do in our working and even our social lives. For example when seeking to improve the performance of a business organization, we seek to identify its critical components and understand how they interact. The same is true in navigating the complex affinities that can arise in our rich network of relationships. Mathematics, in working in an abstract setting, can give us powerful conceptual tools for this type of task.

As a consequence of this, curriculum development work goes into the construction of activities that give students experience at understanding "how things work." Many different labels (constructivism, problem-based or inquiry-based learning) are used to describe various realizations of this activity, but the term "discovery" learning is often used. Unfortunately, it has also borne the weight of much negative criticism, leading to a confused wide-ranging debate often called the "math wars." The problem of course is that discovery takes time and that would appear to be time taken away from the mastery of technical routines arguably of critical importance for the learning of mathematics. There are all kinds of Whiteheadian half-truths here making for an impossibly tangled debate, and I'm not sure that we have even yet properly emerged from the confusion.

One rather legitimate critique of many of the discovery learning activities, that has appeared in books and articles, is that the kids are struggling to "discover" standard items of knowledge that they might more effectively have simply been given.

> Enter a constructivist who says: Michael will have a better relationship with the manipulation of fractions if he discovers the rules himself. So situations are created (often with great ingenuity) that will lead children to "discover" the rules of arithmetic. But being made to "discover" what someone else (and someone you may not even like) wants you to discover (and already knows!) is not Michael's idea of an exciting intellectual adventure. The idea of invention has been tamed and has lost its essence. He wants to fly, but what this kind of constructivism offers him is more like decorating the captive bird's cage [5] (p. 722).

But this does not get at the real problem behind many of the discovery learning examples—*that* lies in the context of the activity itself, in its relevance to the larger life of the student.

> The important difference between the work of a child in an elementary mathematics class and that of a mathematician is not in the subject matter (old fashioned numbers versus groups or categories or whatever) but in the fact that the mathematician is creatively engaged in the pursuit of a personally meaningful project. In this respect a child's work in an art class is often close to that of a grown-up artist. [6] (p. 249).

Watch a group of students engaged in an escape room activity. The message they are struggling to decode will give them the key to unlock the next doorway and that will bring them one step closer to the final chamber. The activities we bring into the math classroom are too often fragments, not part of a meaningful experience.

5. The student experience.

The central idea in John Dewey's writings on education is the quality of the student experience. Of course the experience Dewey refers to is the one that happens at the moment, not some future experience that awaits the student who learns the lesson well.

> I assume that amid all uncertainties there is one permanent frame of reference: namely, the organic connection between education and personal experience. …the trouble is not the absence of experiences, but their defective and wrong character—wrong and defective from the standpoint of connection with further experience [7] (p. 8).

For Dewey, "the central problem of an education based upon experience is to select the kind of present experiences that live fruitfully and creatively in subsequent experiences" [7] (p. 9). Discovering and creating such experiences or activities is indeed the focus of my own curriculum work.

Whitehead [2] also focuses on the immediate experience of the student—he refers to it as "Life" with a capital L.

> There is only one subject-matter for education, and that is Life in all its manifestations. Instead of this single unity, we offer children—Algebra, from which nothing follows; Geometry, from which nothing follows; Science, from which nothing follows; History, from which nothing follows; a Couple of Languages, never mastered; and lastly, most dreary of all, Literature, represented by plays of Shakespeare, with philological notes and short analyses of plot and character to be in substance committed to memory. Can such a list be said to represent Life as it is known in the midst of the living of it? [2] (pp. 6-7)

Here Whitehead is clearly not denigrating Algebra or the plays of Shakespeare, but he despairs of the narrow technical version that typically dominates the classroom. Whitehead certainly understands the critical role that technical mastery plays in the learning of mathematics and indeed in any creative enterprise, but it must be properly situated in what he calls the Rhythm of Education [2] (Chapter II). Here he identifies three stages of learning: Romance, Precision and Generalization. To some extent all our learning proceeds by passing through each of these stages in order, such that, roughly speaking, the child is dominated by Romance, the youth by Precision, and the adult by Generalization. In practice however the stages cycle continuously like eddies in the fast flowing stream of life (and indeed at different times we can all be children or adults).

The first stage is one of ferment, novelty and mystery, of hidden possibilities and barely justifiable leaps. This stage, in its fullness, motivates the second stage in which we strive for comprehension and mastery—ideas must be tamed and organized, requiring care, honesty and restraint. Finally, the third stage is essentially a return to Romance, but now with the technique acquired at stage two. Our ideas have new power because we have harnessed them. The great fruit of this ultimate stage of learning is wisdom: the capacity to handle knowledge. The central point that Whitehead makes is that the discipline of stage two must not be imposed until the fullness of stage one has properly prepared the student. Failing that, the knowledge that is obtained will be inert and ineffective. That seems often to be the case for the knowledge that students bring into my first-year university course.

Papert also thinks at length about the experience of the student and builds his classroom around what he calls projects.

> This *project-oriented* approach contrasts with the *problem* approach of most mathematics teaching: a bad feature of the typical problem is that the child does not stay with it long enough to benefit much from success or from failure. Along with time scale goes structure. A project is long enough to have recognizable phases—such as planning, choosing a strategy of attempting a very simple case first, finding the simple solution, *debugging it* and so on. And if the time scale is long enough, and the structures are clear enough, the child can develop a vocabulary for articulate discussion of the process of working towards his goals [6] (p. 251).

Barabe and Proulx call Papert's project-oriented approach a complete rebuild, "une reconstruction complète" of school mathematics [8] (p. 26), defining the mathematics curriculum itself not in terms of content but as the activity of the students.

6. A complete rebuild

Is our current school mathematics curriculum amenable to the kind of rebuild that might be needed to bring it in line with the pedagogical model inspired by Dewey's Experience, Whitehead's Rhythm, and Papert's Projects? Some decades ago, Papert's answer to this question is that it would require considerable creative work.

> Is it possible for children to do creative mathematics (that is to say: to *do* mathematics) at all stages of their scholastic (and even adult!) lives? The author will argue that the answer is: yes, but a great deal of creative mathematical work by adult mathematicians is necessary to make it possible. The reason for the qualification is that the traditional branches of mathematics do not provide the most fertile ground for the easy, prolific growth of mathematical traits of mind. We may have to develop quite new branches of mathematics with the special property that they allow beginners more space to romp creatively, than does number theory or modernistic algebra. In the following pages will be found some specific examples which it would be pretentious to call 'new pedagogical oriented branches of mathematics' but which will suggest to co-operative readers what this phrase could mean [6] (p. 250)

The examples Papert had in mind were realized through technology, and in this he was much ahead of his time as the ideas that drove his Logo software are, decades later, being piloted in today's elementary classrooms. Technology certainly facilitates much of the current work in curriculum renewal, but the fundamental driver of today's curriculum renewal is, for most mathematicians, not technology, but "activities that provide fertile ground for the easy, prolific growth" of what we now call "mathematical thinking." Papert is correct that not all branches of mathematics nurture this well, but things in the mathematical world have changed enormously since 1972, giving us dynamic new branches and even the "traditional branches" have new foliage and brighter, more sophisticated blossoms. The mathematics needed for this rebuild is there, ready and waiting.

The problem is that the decision to structure the curriculum around the development of technical skills has effectively shut out Dewey's experience and Whitehead's Life. Let me give an example with an activity taken from the Ontario Grade 12 Advanced Functions curriculum.

Ontario Grade 12 Advanced Functions
3.3 Solve problems, using a variety of tools and strategies, including problems arising from real-world applications, by reasoning with functions and by applying concepts and procedures involving functions (e.g., by constructing a function model from data, using the model to determine mathematical results, and interpreting and communicating the results within the context of the problem).

Sample Problem. The pressure of a car tire with a slow leak is given in the following table of values:

Time, *t* (min)	**Pressure, *P* (kPa)**
0	400
5	335
10	295
15	255
20	225
25	195
30	170

Use technology to investigate linear, quadratic, and exponential models for the relationship, of the tire pressure and time, and describe how well each model fits the data. Use each model to predict the pressure after 60 min. Which model gives the most realistic answer? [9] (pp. 97-98).

It turns out that I actually generated this very data set in 1998 in a Grade 12 class together with my then PhD student Nathalie Sinclair, but the original activity was richer in an essential way than the Ministry curriculum description would suggest. First of all the data was taken by the students themselves from a real tire. Thus they were able to feel the jet of air pushing against their finger and that reinforced their view that there was a force, that they called pressure, pushing the air out. As a result when we challenged them to build a model, they struggled as this "pressure" seemed to them to be a complicated phenomenon. But because of that they were ripe to be surprised [10]—and we suggested that it was perhaps not complicated at all, that the molecules of air were simply scooting around the inside of the tire, bouncing off whenever they hit the inner surface of the tire, except those rare cases in which they encountered the hole instead and then then they passed right through into the outside air. That "pressure" against the finger was simply the normal punch of the speeding molecules. With that memorable insight, the students were ready to observe that the flow rate must be proportional to the number of molecules inside the tire, leading directly to the exponential model. Rather than trying to fit the data with different models, the students were being led to proceed exactly as would a mathematician––asking first of all the mechanistic question: "what exactly is going on here?"

Thus, the Advanced Functions formulation, in beginning with the technical agendum—"let's practise finding the best-fit curve"—betrays the students and robs them of the essential mathematical experience. We need a new curriculum structure that honours that experience [11-12].

That brings us to the ultimate question, the one Dewey called "the central problem of an education based upon experience" [7] (p. 9)—what counts as an authentic experience?

7. Aesthetics and integrity

In his remarkable book, *Art as Experience*, John Dewey constructs an entire theory of the aesthetic around individual experience; in effect he calls us to be artists in all our interactions, and the canvas upon which we paint is our very experience.

His main thesis is that the aesthetic experience is jointly constructed between painter and viewer, between performer and audience, that both are called to be artists in a shared experience.

> The word "aesthetic" refers, as we have already noted, to experience·as appreciative, perceiving and enjoying. It denotes the consumer's rather than the producer's standpoint. It is Gusto, taste; and, as with cooking, overt skillful action is on the side of the cook who prepares, while taste is on the side of the consumer, as in gardening there is a distinction between the gardener who plants and tills and the householder who enjoys the finished product [13] (p. 37).

For me this captures the essential character of the teacher-student relationship and I take Dewey's concept of the aesthetic as the authoritative principle guiding mathematics curriculum design. In fact, I add to this a closely related principle, that of integrity. For me these are two aspects of human activity that are universal in their authenticity and relevance to the lives of our teachers and our students. They are experiential versions of the simpler ideas of beauty and wholeness—I take aesthetics to be the *experience* of beauty and integrity to be the *experience* of wholeness. In these extensions I emphasize the organic connection between the object and the experience of the beholder, between the activity and experience of the participant. Artists and craftsmen alike seek to infuse their work with *experiences* of beauty and wholeness, and we teachers of mathematics should do no less.

> Experience in this vital sense is defined by those situations and episodes that we spontaneously refer to as being "real experiences"; those things of which we say in recalling them, "that was an experience." It may have been something of tremendous importance--a quarrel with one who was once an intimate, a catastrophe finally averted by a hair's breadth. Or it may have been something that in comparison was slight--and which perhaps because of its very slightness illustrates all the better what it is to be an experience. There is that meal in a Paris restaurant of which one says "that was an experience." It stands out as an enduring memorial of what food may be. Then there is that storm one went through in crossing the Atlantic—the storm that seemed in its fury, as it was experienced, to sum up in itself all that a storm can be, complete in itself, standing out because marked out from what went before and what came after…In such experiences, every successive part flows freely, without seam and without unfilled blanks, into what ensues [13] (p. 43).

This excerpt makes it clear that integrity is an integral part of Dewey's concept of the aesthetic, and this is no doubt the case for any artist, but for emphasis I choose to offer it separately as the seamless flow seems to me to be a particularly important aspect of curriculum.

Why should teachers of mathematics be concerned with these aesthetic aspects of experience? There are many answers. At the highest level, it is because these qualities are what makes us human and mathematics is a profoundly human endeavour. More specifically these qualities infuse every aspect of a mathematician's life [10]. Nathalie Sinclair's marvelous book *Mathematics and Beauty* [14] discusses the different ways in which aesthetic considerations impact the mathematician's work, the motivational—

what structures are worth investigating, the generative—how we come to understand the workings of the structure, and the evaluative—choosing the best among different possible approaches or analyses. Of these, it is the generative role that has always been the most fascinating to me, how beauty and wholeness can be a reliable guide in our search for the correct path. As one who has studied the ideas of evolutionary biology, I find it fascinating to wonder about the ways in which our allegiance to beauty and wholeness might have evolved. Poincare [15] famously wrote about this asserting that this link worked through the unconscious and was fundamental to the discovery process. Papert in a wonderful essay [16] took this theme up in his quest to use technology to show the process at work in the activity of children.

8. Art and Science.

I have been treating mathematics as a creative art, but the discussion above suggests that art is closely aligned with science—or if you like, beauty is closely aligned with truth.

In an interesting and little known essay, Geoffrey Vickers observes that the loss of an organic connection between Science and Art is recent and unnatural. He bemoans

> the sad history of Western culture which, over the last two centuries, has so narrowed the concepts of both Science and Art as to leave them diminished and incommensurable rivals,—the one an island in the sea of knowledge not certified as science; the other an island in the sea of skill not certified as art… Moreover the two words "Ars" and "Scientiae" not only embraced virtually all skill and knowledge, but also overlapped each other's territory without offense [17] (p. 143).

To bring us back to our myth, we see this sad disconnect more clearly than anywhere else in the structure of the secondary mathematics curriculum. Here, other than in the hands of an exceptional teacher, what little sense of beauty might have survived the mathematical journey through elementary school, is trodden into the ground in the grim technical race to prepare the students for a STEM future in a technologically-driven society. The irony is that employers who are hiring in the STEM disciplines are now more interested in so called secondary traits, the five C's, creativity, critical thinking, collaboration, cooperation and care [18].

9. Summing up

> This wide gulf between real mathematics and school mathematics is at the heart of the math problems we face in school education. I strongly believe that if school math classrooms presented the true nature of the discipline, we would not have this nationwide dislike of math and widespread math underachievement [1] (pp. 22-23).

In this challenging assertion, Jo Boaler indicates clearly the direction in which we need to move and asserts that this has the power to fix the problems that have plagued mathematics education for decades. There are many teachers who would wholeheartedly agree. Indeed the global response many years ago to Lockhart's *Lament* [19] testifies to that. But from recent encounters with officials from the Ontario Ministry of Education I have seen no indication that future curriculum revisions might move in this direction. Indeed, even the teachers, who typically understand the need for a richer curriculum, are hesitant to try new "experiences." They automatically class these as "enrichment," and while these are worthwhile, they can only be accommodated if there is time, if the mandated technical ground has been covered. The idea that these experiences *are* the curriculum and that the technical ground can safely be left to fend for itself, that it might even be more fertile under the protective blanket of a rich environment, is quite a new and even intimidating idea.

There is one piece of the myth that I have not directly addressed and that is whether these mathematical experiences really are accessible to all our students. Sinclair asserts that students at all ages are clearly aesthetic beings and the creative dimension of their experience in school is significant for them [14] (pp. 114-116). Indeed but we are always aware of the apparently huge diversity among our students in their capacity to handle sophisticated stories. That is exactly the reason to work with "complex mathematical ideas using a) a low mathematical floor, requiring minimal prerequisite knowledge, and b) a high mathematical ceiling, offering opportunities to explore more complex concepts and relationships and more varied representations" [10] (p. 236). Boaler also discusses such activities.

> Low floor, high ceiling tasks allow all students to access ideas and take them to very high levels. Fortunately, low floor, high ceiling tasks are also the most engaging and interesting math tasks with value beyond the fact that they work for students of different prior achievement levels…Such teaching, though demanding, is also extremely fulfilling for teachers, especially when they see students who lack confidence and were previously low-achieving take off and soar [1] (p 115).

Of course these notions intersect Whitehead's Romance and Precision. Indeed an aesthetic experience will work at different levels in different students. Some will be ready and eager to roll it right up onto the stage of Precision; others who might not yet have the right analytical tools can still play with it and wonder. Indeed "wonder" is a magical word and Sinclair and Watson [20] have a marvellous book review in which they play with its two shades of meaning.

Indeed, all our students, no matter how deprived their past, can learn to wonder at beautiful things. Given this, the teacher's responsibility is to share her own aesthetic experience with her students. We all know that students respond positively to such sharing: "I bring this problem to you because it is important to me, because I love it." We need to trust that if our collection of works is rich enough in terms of breadth and sophistication, and if we do justice to the technical skills required for each work, then by the end of the K-12 journey, they will be ready for whatever challenge tertiary mathematics might throw at them.

Last week I brought a recently developed unit [21] into a 2-hour grade 11/12 class. The objective was to understand why we have 12 notes in an octave, indeed, what remarkable property of the number 12 is at play here (why not 10 or 15?). We ranged over a broad array of activities, involving frequencies, harmonies, and the nature of perception—how good are we at distinguishing notes of slightly different frequencies? The answer is that we are amazingly good. Indeed, using a tone generator the students discovered that the brain has the power to distinguish between an eardrum vibrating at 2000 oscillations per second and one with a vibration of 2010. The more you think about that the more extraordinary it seems to be. That's experience. That's Life! Our students are hungry for it.

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