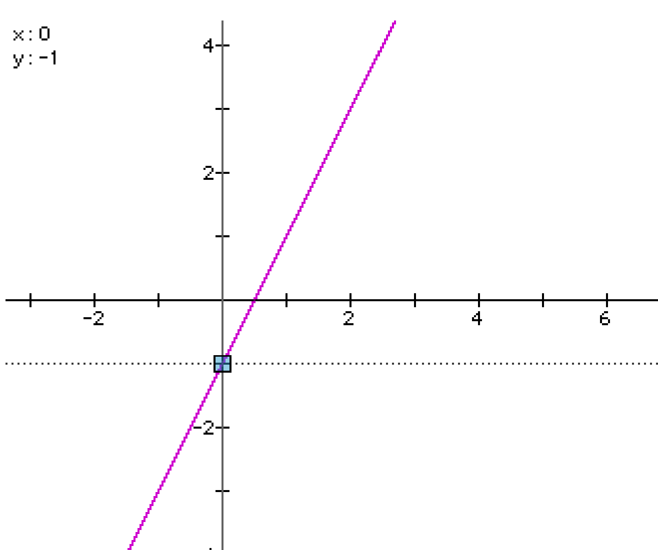
So, you're having trouble with linear functions. Well, sit back, relax, return your trays to their upright position and fasten your seat belts We're about to fly to the world where linear functions are King. Where points, slopes and equations all come together in a devine, coherent, almost epiphenical way.

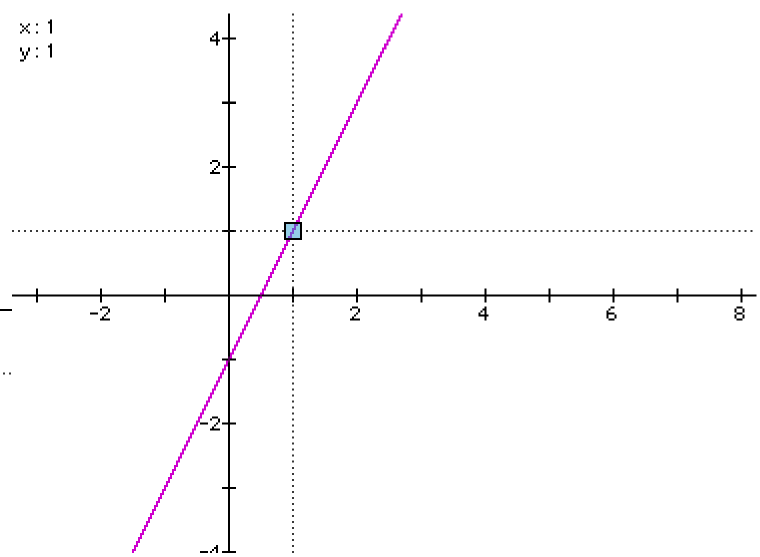
I could go on, but I'm sure you'd like me to get to the point......How's about (0,-1)? or (4, 7)? Just a little bit of math humour...sorry, I couldn't resist.

If you know one point on a line and you know the steepness(slope) of the line then you have enough information to determine an equation for the line which you could use to determine any point on the line.
"*What did he say?*"

Too many words? OK. Let's try some pictures and words:

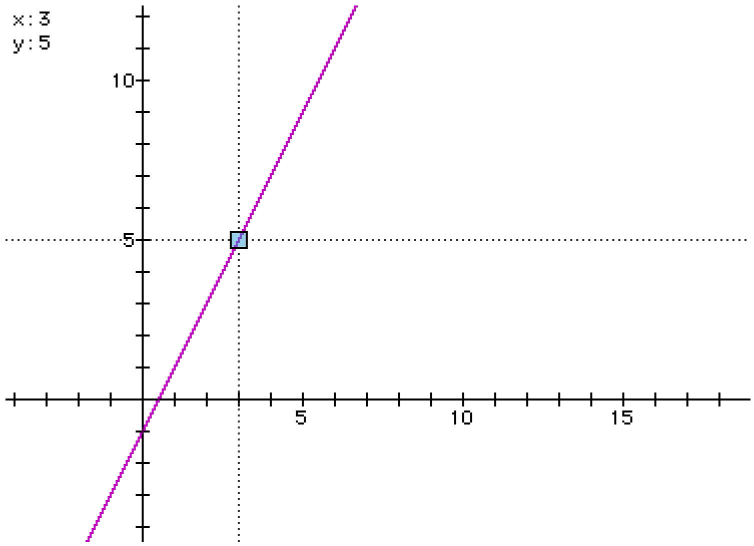


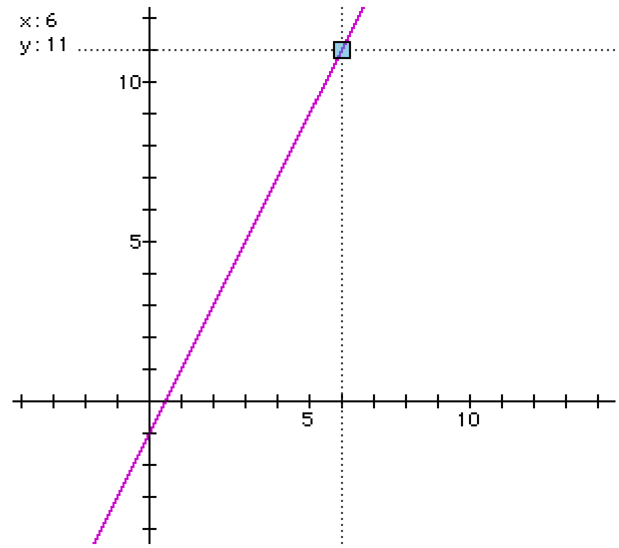
Here's a line that has a y value of -1 when x is 0.



Here's the same line that has a y value of 1 when x is 1.

Notice, as x increased by 1, y increased by 2. If you chose any two points on a straight line and divided the change in the y value by the change in the x value you would get the same answer each time. In this case the change in the y value divided by the change in the x value was 2.





Again, we have the same line in both cases.
The slope of this line, which is a mathematical measure of its steepness is determined by dividing the change(Δ) in the y value by the change in the x value:

$$
\begin{aligned}
Slope &= \frac{\Delta y}{\Delta x} \\
&= \frac{11-5}{6-3} \\
&= \frac{6}{3} \\
&= 2
\end{aligned}
$$

When there is no context given then the steepness is called the slope.
However, if the context is motion then the steepness of the line is the object's speed. In the above example the y values could represent distances(measured in meters) and the x values could represent times(measured in seconds).
Then the speed of the object would be its change in distance divided by its change in time.

So...

$$
\begin{aligned}
Speed &= \frac{\Delta distance}{\Delta time} \\
&= \frac{11m-5m}{6s-3s} \\
&= \frac{6m}{3s} \\
&= \frac{2m}{s}
\end{aligned}
$$

We'd say the object's speed was 2 meters per second. That means with the passing of each second the object will gain 2 meters. The letters in the solution represent **m**eters and **s**econds.

Now try a few yourself. Show all the steps.

1. Determine the slope of the line between the following pairs of points:
   a) (3, 8) and (5, 14) b) (4, 12) and (9, -8) c) (-2, -3) and (4, 9)

[Answers] [3] [-4] [2]

2. Detemine the speed of the object given the following pairs of times(h) and distances(km).
   a) (0, 0) and (6, 150) c) (3, 200) and (6, 470) c) (0, -20) and (4, 56)

[Answers] [25km/h] [90km/h] [19km/h]

That wasn't so bad, was it?

Referring back to those graphs that had a slope of 2, we could use this value and any one point on the line to find a bunch of other points. Here's a chart of a sequence of points on that line:

x	0	1	2	3	4	5	6	7	8
y	-1	1	3	5	7	9	11	13	15

You may remember that we learned a formula that we could use to determine the equation of a line if we had the line's slope and one point:

$$y = y_1 + (x - x_1)m$$

When working with linear functions the letter *m* is used to represent the slope. The symbols x_1 and y_1 represent the x and y coordinates of a point on the line.

So, if you wanted to determine the equation of the line that would be created if you plotted these points above then you would use the slope(m) value of 2 and any point. Keep in mind that it doesn't matter which point you choose, the equation will be the same.

Here I'll determine the equation of the line using different points:

(x_1, y_1) = (0, -1) m = 2

$$y = -1 + (x - 0)2$$
$$y = -1 + (x)2$$
$$y = -1 + 2x$$

(x_1, y_1) = (3, 5) m = 2

$$y = 5 + (x - 3)2$$
$$y = 5 + 2x - 6$$
$$y = 5 - 6 + 2x$$
$$y = -1 + 2x$$

You may find it easier to apply distributive property when the 2 is in front of the (x-3). Since the order in which you multiply doesn't matter then you can do that, just as I've done in the next solution.

(x_1, y_1) = (7, 13) m = 2

$$y = 13 + (x - 7)2$$
$$y = 13 + 2(x - 7)$$
$$y = 13 + 2x - 14$$
$$y = 2x - 14 + 13$$
$$y = 2x - 1$$

This may appear different than the previous two solutions, but the order in which you add doesn't matter.
2x-1 is the same as -1+2x

3. Now try some on your own. Use the highlighted points from the chart and show all your work.

x	0	1	2	3	4	5	6	7	8
y	-1	1	3	5	7	9	11	13	15

For those of you who were hoping for an in-flight movie right about now, sorry, our DVD player broke during our last flight. How about a bit of comic relief? I've got one....this one is my daughter's favourite.

"Why does the sea roar?"
If you had crabs on your bottom you'd roar too.

Let's get back to work....

"If you know one point on a line and you know the steepness(slope) of the line then you have enough information to determine an equation for the line which you could use to determine any point on the line."

Remember this quote from page 2? We've now managed to find an equation of a line, but it's no good to us unless we do something with it. Well if we know the x coordinate of a point on the line we could find its corresponding y value. Likewise, if we know the y coordinate of a point we could find its corresponding x value......blah, blah, blah.....Is this what you're saying to yourself right now? If it is then this could be part of the reason why you've been having difficulty with this topic. Remembering and understanding statements like the one just made are just as important as acquiring the technical skills of algebra. Without them, all the algebraic expertise in the world is not going to help you solve a problem. OK, I've preached enough...time to get back to the matter at hand before I lose you to those in-flight radio stations.

If you're like me then you may be finding all this x and y stuff kinda dull. When I was in school I found math much more interesting when we were solving an applied problem....when there was some context being considered. The best context that I think we can use to help us understand these types of functions is motion. So, here's a problem:

A cyclist is riding at 30km/h. After 2 hours of cycling the cyclist was 80km from home.

4. Determine an equation that will give you the cyclist's distance(d) from home at any time(t)

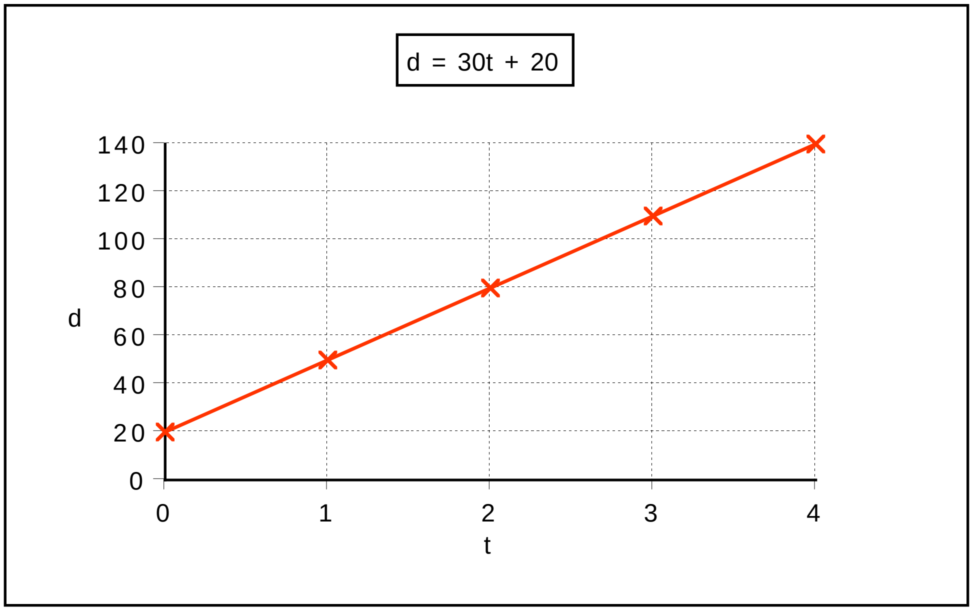
Solution: we can use the formula we learned earlier, but just adapt it for this context.

$$y = y_1 + (x - x_1)m \quad \text{--->} \quad d = d_1 + (t - t_1)s$$

Here, the letter *s* is used to represent the speed. The symbols t_1 and d_1 represent the distance(d_1) the cyclist is from home at a particular time(t_1).

So, $d = 80 + (t - 2)30$. Showing all the steps, simplify this equation.

You should have gotten $d = 30t + 20$. Those who have been thinking about their answer may have noticed that at time t= 0 hours the distance from home was not 0 km. This cyclist was 20 km from home when he/she started their journey. Remember all those race questions we solved? This 20 represents a head start. Notice how this equation "looks" when you graph it:



4. The following data was used to create the graph above. Explain why, in this situation, a straight line can be drawn through the plotted points

t	d
0	20
1	50
2	80
3	110
4	140

____

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____

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5. Now use the equation to find the distance the cyclist is from home after 7 hours of cycling.

Solution: using the formula all we need to do is substitute the time of 7 hours into the t value in the equation then solve.

$d = 30t + 20$

$d = 30(7) + 20$

$d = 210 + 20$

$d = 230$ Therefore, after 7 hours the cyclist is 230 kilometers from home.

Now it's your turn, find the cyclist's distance from home after 24 hours of cycling.

I'm sure you were able to determine that the cyclist was 740 km from home after 24 hours of cycling. I don't know what you think, but I think that was one heck of a ride. Pretty unrealistic eh? Well, maybe not so unrealistic after all. I just read in the paper today that a phys ed teacher, who is also an ultra marathon runner, is going to attempt to run on a treadmill in a health club for 24 hours continuous. Only stopping to pee.

FYI, you can also use the equation to find the time the cyclist reaches a particular distance.

6. Determine the time the cyclist is 410 kilometers from home.

Solution: You solve this similar to the way you solved question 5, but a bit more care is required . Substitute the distance of 410 km into the d value in the equation then solve.

$d = 30t + 20$

$410 = 30t + 20$

$410 - 20 = 30t + 20 - 20$ What you do to one side you must do to the other side.

$390 = 30t$

$\frac{390}{30} = \frac{30t}{30}$ Again, what you do to one side you must do to the other side.

$13 = t$ Therefore, when the cyclist is 410 kilometers from home he/she's been cycling for 13 hours.

Now it's your turn, find the time the cyclist reaches the following distances from home.

	a)	b)	c)
	d = 575 km	d= 327.5 km	d= 833 km
[Answers]	[t = 18.5 h]	[t = 10.25 h]	[t = 27.1 h]

I don't know about you, but these small seats on these charter flights are beginning to cramp my legs. It's probably time to take a break.

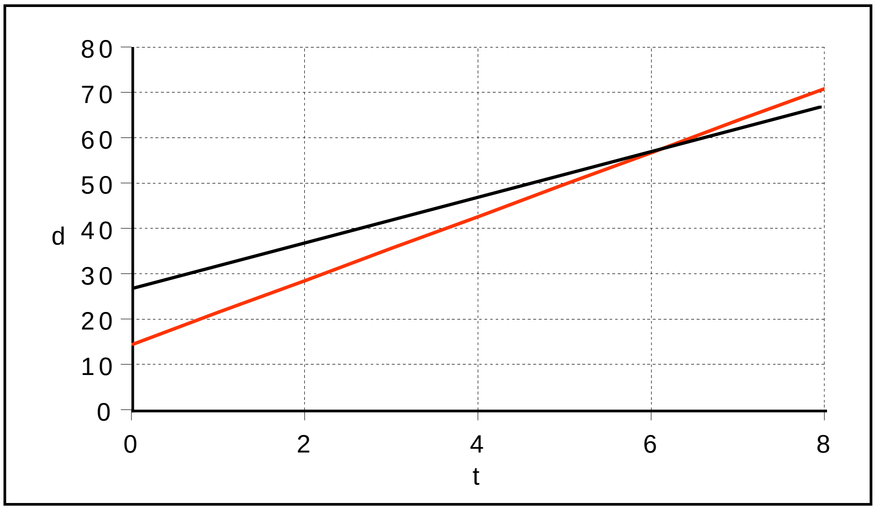
I'm sure by now you've realized I'm not going to let you get away without taking another look at the Big Question. But this time we're going to give Alan and Shamita a break. Ultimately in solving the Big Question we learned how to find the place where two lines cross.

7. Two runners' distances from a starting line are given by the following equations:

$$d = 7t + 15 \qquad\qquad d = 5t + 27$$

Find the time when the runners' distances from the starting line are the same. Distances are measured in meters and times in seconds

Solution: Before we jump in with both feet we should see what the situtation looks like.



Where the lines intersect they have one point(t, d) in common.
It looks as if the time when their distances are the same is about 6 seconds.
That distance lies somewhere between 50 and 60m

To determine the exact time and distance we need to do some algebra.

Since their distances(d) are the same where the lines cross then

$$d = 7t + 15 \qquad\qquad d = 5t + 27$$

$$7t + 15 = 5t + 27$$

Now we need to solve this equation for t. Remember we need to maintain equality at each step so we need to do the same thing to both sides of the equation when we are isolating t.

$$7t + 15 - 5t = 5t + 27 - 5t$$

$$2t + 15 = 27$$

$$2t + 15 - 15 = 27 - 15$$

$$2t = 12$$

$$\frac{2t}{2} = \frac{12}{2}$$

$t = 6$ The runners' distances are the same at 6 seconds.

To find the distance substitute the t value into both equations. What do you notice?

Soooooo, you thought you were getting off easy. Well, we haven't landed yet, but we have begun our decent. You'll notice that the captain has turned on the no smoking signs. Please return your trays to their upright position and fasten your seat belts. This could get bumpy. Hopefully you haven't forgotten those skills you acquired when we were cruising at 30 000 feet.

1. The following pairs of ordered pairs represent a cyclist's time and distance from home. Determine the cyclist's average speed. Indicate whether the cyclist is riding away from home or towards home.
   a) (0, 0) , (3, 93)   b) (3, 93) , (7, 229)   c) (7, 229) , (9, 149)

2. Find the slope of the line between the following pairs of ordered pairs.
   a) (8, 15) , (11, 33)   b) (11, 33) , (26, 33)   c) (26, 33) , (29, -7)

3. A runner's average pace is 14km/h. After 2 hours the runner was 39 km from her home.
   a) Determine an equation that represents the runner's distance at any time.
   b) How far from home was the runner when she started?
   c) Use the equation to determine where the runner was after 5 hours of running.
   d) Use the equation to determine when the runner was 50 km from home.

OK passengers, our decent is about to get a little bumpy.

4. Consider the following points: (3, 7) and (-4, 7/3).
   a) Find the slope of the line between the points. (leave your answer as a fraction)
   b) Find the equation of the line through the points. (leave the slope as a fraction)
   c) Use the equation to determine y when x is 12.
   d) Use the equation to determine x when y is 1/3.

5. Consider the following equations: Jack: $d = 8t - 9$ and Jill: $d = 6t + 12$
   a) Sketch both lines on the same axes.
   b) Determine the time(seconds) when both distances(meters) are the same.
   c) Which runner is farther ahead when t is 14 seconds?
   d) Determine the distances between the runners when t is 14s.

We're about to encounter some turbulence. All flight attendants please take your seats.

6. Consider the following equations. $y = -3x + 12$ and $y = (16/3)x - 2$
   a) Sketch both lines on the same axes.
   b) Determine the x value when both y values are the same.
   c) What are the y values when they are the same?

On behalf of the captain and the crew we thank you for flying Linear Airlines. We hope you enjoyed your flight and look forward to seeing you again.