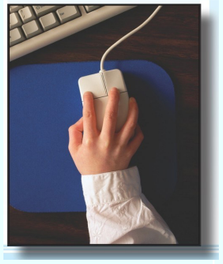
TOOLS, COMPUTER SOFTWARE AND TECHNOLOGY

- Are central to instruction and are used to improve and deepen understanding;
- Are used to discuss mathematical relationships, concepts and ideas; and
- Are used to provide a model for complex concepts.
- Provide opportunities for understanding, consolidation and practice;
- Are used by students to solve problems and apply skills; and
- Enable students to develop communication in mathematics through visual representation.

COMPONENTS	K-2	3-5	6-9	10-12
Calculators	• 1 line display calculators (TI 08)	• 2 line display calculators (TI 10 and TI 15)	• 2 line display calculators (TI 15)	• Graphing calculators, Nspire and peripherals
Manipulatives	• 2 line display calculators (TI 10)	• Students use manipulatives and technology as tools for learning	• Graphing calculators (TI 83+/84)	• Students use manipulatives and technology as tools for learning flexibly and innovatively
Computers	• Students explore and use manipulatives and technology as tools for learning	• Computer programs and applications	• Students use manipulatives and technology as tools for learning with flexibility	• Computer programs and applications (e.g., spreadsheets)
Interactive White Boards				• Dynamic geometry software
Student Response Systems	• Computer programs and applications	• Dynamic geometry software	• Computer programs and applications (e.g., spreadsheets)	• Dynamic statistical software
Document Cameras		• Dynamic statistical software	• Dynamic geometry software	• Computer algebra systems
Assistive Technology		• Internet	• Dynamic statistical software	• Internet
e-Texts			• Internet	

CONTACT INFORMATION

For further information, please go to TDSBweb and see Program Coordinator, Mathematics, under ***Contact Us***

Toronto District School Board

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MATHEMATICS/NUMERACY

K -12

TEACHING AND LEARNING

EXPECTED PRACTICE SERIES

Mathematics

reveals patterns, order and relationships which help us to understand the world around us. It is an increasingly varied discipline that deals with number, spatial sense, data, measurement, and observations requiring inference, deduction, modeling and communication. *Numeracy* is the application of mathematical concepts and procedures in a variety of contexts. It is interdisciplinary as it uses mathematics as a tool to explore problems and situations throughout the curriculum and every day life.

The Toronto District School Board identifies the teaching and learning of mathematics as a priority and is committed to providing support so that all learners achieve the highest academic levels.

(Taken from *TDSB Mathematics Foundation Statement P.004 CUR*)

The Effective Mathematics Classroom Environment

- Both teachers and students demonstrate a positive attitude towards mathematics
- Is a safe, supportive and respectful classroom
- Includes engaging, challenging and authentic mathematics
- High expectations are held by teachers and students
- Provides collaborative learning opportunities
- Student mathematical thinking and learning is evident and posted
- Student generated work and/or co-constructed with the teacher are posted and clearly defined
- Students have easy access to and choice of learning tools, mathematics resources and technologies
- Includes activities and resources that are inclusive and reflect the needs of students with varying backgrounds,abilities, interests and learning styles

Professional Learning in Mathematics

Teachers engage in collaborative professional learning, such as:

- Co-planning
- Co-teaching
- Lesson study
- Inquiry
- Webinars
- Modular learning
- Self-directed Professional learning communities
- Teacher moderation
- Professional Learning Teams
- Critical Friends
- Book Study
- Demonstration Classroom visits

Teachers study relevant topics such as:

- Engaging students in relevant challenging mathematics;
- Current pedagogy in mathematics e.g. three-part lesson, teaching through problem-solving, Bansho, Congress, Gallery Walk;
- Math content for teaching;
- The developmental continuum of math concepts and skills;
- The use of the technology in teaching and learning of mathematics;
- Effective assessment tools and strategies for mathematics teaching & learning

The Learning Destination for All Students in Mathematics

Successful planning and teaching enables students to consistently and independently

- Understand, remember, and apply mathematical concepts
- Articulate clear understanding of mathematical concepts and ideas and give clear examples of use
- Apply concepts, skills, and strategies to propose solutions to problems
- Analyze problems, use a variety of strategies & processes to find solutions, and be able to check and evaluate effectiveness of processes used
- Work effectively alone and with others
- Communicate effectively using words, symbols, and representations
- Connect ideas to self, others and other ideas/tasks
- Use "mathematical habits of mind", for example persistence, questioning, prior knowledge/experience, precision of language and thinking
- Demonstrate learning and thinking in a range of ways including:
  - Products e.g. work samples, tests, quizzes;
  - Observations e.g. class work, demonstrations, performance tasks, teacher observations; and
  - Conversations e.g. discussions, written reflections, journal entries, conference, interview.

(Adapted from Reporting in Mathematics 2010, Ann Davies)

Characteristics of Mathematical Literate Students

- Can communicate their mathematical thinking and can understand the mathematical reasoning of others
- Have a sense of numbers and are sufficiently efficient in their work that their thinking builds as they progress towards a solution
- To make sense of mathematics
- Make connections between concepts and see pattern throughout mathematics
- Are willing to persevere in order to understand and solve mathematical problems
- Are developing depth and flexibility in their thinking

(The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario)

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COMPREHENSIVE MATHEMATICS PROGRAM

- Is focused on having students make sense of mathematics;
- Is based on problem solving and investigation of important mathematical concepts;
- Begins with the students' understanding and knowledge of the topic;
- Includes students as active, rather than passive participants in their learning;
- Allows students to engage in a variety of problem solving experiences to develop conceptual and procedural understanding and skills through exploration, investigation, direct instruction and practice;
- Has students communicate and investigate their thinking through ongoing discussion;
- Includes all students, whether in the choice of problems or in the communicating of mathematical ideas;
- Incorporates ongoing assessment of student understanding to guide future instruction;
- Allows students to develop their ability to ask questions and to plan investigations to answer these questions;
- Enables students to spend most of their time working in partners or small groups to explore and learn concepts, complete tasks, represent, justify and consolidate their thinking;
- Engages students in investigations or questions prompted by literature or a topic in other subject areas;
- Includes opportunities for students to apply math knowledge and skills in other subject areas;
- Integrates parents/caregivers as partners in supporting students disposition and achievement;
- Uses optimal groupings to encourage collaborative discourse and critical thinking.

INSTRUCTIONAL STRATEGIES IN MATHEMATICS

- Are used within a three part lesson framework;
- Are based on a problem solving or investigation approach;
- Include a balance of instructional methods including play, exploration, investigation, direct instruction, and practice;
- Incorporate effective Literacy strategies;
- Promote students thinking about the big ideas in mathematics through an inter-strand approach;
- Are set within a rich context;
- Allow students to communicate and justify thinking and strategies;
- Allow for differentiated learning within flexible groupings.

Strategies for Thinking and Consolidation

Gallery Walk	Math Congress	Bansho
• Students and teachers circulate to examine student recorded solutions to a lesson problem • Students read solutions and give oral and written feedback • Teachers assess the range of mathematical thinking in the different solutions • Teachers use the information to determine the focus of consolidation and next steps for planning	• Two or three student solutions are used to conduct a whole class discussion that will develop every student's mathematical thinking • Teachers use student solutions to prompt them to reason about big math ideas • Specific ideas and strategies are generalized and connections made to previous math discussions and learning • Students defend and support their solutions and thinking • Teachers use questioning to prompt all students to reason and generalize based on the lesson goals	• Means board writing in Japanese • Math thinking is organized and recorded as it occurs in the lesson • Mathematical expressions, diagrams, solutions and strategies are recorded for all to engage in • Various solutions are organized and compared through teacher and student questioning • Students deepen their mathematical thinking through comparing generalizing and summarizing

Adapted from: Communication in the Mathematics Classroom, Special Edition #13 Literacy and Numeracy Secretariat

CONTINUED

ASSESSMENT FOR AND AS LEARNING

COMPONENTS	DESCRIPTOR	LOOK FORS
Teacher Moderation	• An examination of student work with colleagues to compare interpretations of student results, and confirm judgments about levels of achievement.	• Professional learning is focused on student work and taking student(s) to the next level of learning; • Structural regular opportunities to examine student work collaboratively in the classroom setting; • Calibrate standards of expectations and practice among teachers within and across grades; • Develop school-wide beliefs, values and practices that support students' well being and success; and • School staff can explain what is the difference that makes the difference for their students.
Peer and Self Assessment	• Assessment of student work or learning processes by self or classmates using the established success criteria.	• Students have regular and structural opportunities to speak to each other about their progress and their work; • Students can describe what success looks like based on set criteria; and • Students have a clear understanding and can articulate why they are learning, what and how they are learning.

COMPONENTS	K-2	3-5	6-9	10-12
Assessment for and As Learning **Data Collection Methods**	• Learning Goals • Success Criteria • Descriptive feedback • Student interviews • Observational data • Student work (oral, written representations) • Student models using manipulatives	• Learning Goals • Success Criteria • Descriptive feedback • Student interviews • Observational data • Student work (oral, written) • Student models using manipulatives • Exit Cards • Reflective Journals • Peer and self assessment items • Portfolio • Peer editing	• Learning Goals • Success Criteria • Descriptive feedback • Student interviews • Observational data • Student work (oral, written) • Student models using manipulatives • Exit Cards • Reflective Journals • Peer and self assessment items • Portfolio • Peer editing	• Learning Goals • Success Criteria • Descriptive feedback • Student interviews • Observational data • Student work (oral, written) • Student models using manipulatives • Exit Cards • Reflective Journals • Peer and self assessment items • Portfolio • Peer editing

ASSESSMENT FOR AND AS LEARNING

- ongoing assessment
- demonstrates what students already know
- informs instruction that is differentiated and personalized
- monitors students progress
- ongoing self and peer assessment
- provides feedback from students to other students
- informs students of their own learning styles/preferences
- allows for individualized goal setting

COMPONENTS	DESCRIPTOR	LOOK FORS
Student Work	• Through facilitated discussions with students and teacher moderation of student work, teachers assess student learning, plan and provide feedback, and adjust instruction.	• Student work used to share strategies and solutions, consolidate learning, prompt mathematical communication; • Student work samples guide professional learning and decision making.
Learning Goals	• What students should be able to do by the end of a period of instruction (e.g. a lesson, series of lessons, or subtask).	• Founded on overall curriculum expectations and big ideas in mathematics; • Stated in clear language.
Success Criteria	• A description of successful attainment of the learning goals that shows what success "looks like". • Success criteria are used to guide descriptive feedback that informs the next level of teaching and learning.	• Co-constructed between teachers and students; • Observable in the classroom; • Accessible to students for reference and scaffolds their learning and why; • Describes what success looks like without showing students which solutions or strategies to use; • Based on expectations for performance as stated in all four areas of the achievement chart.
Rich Assessment Task	• Authentic activity, exercise, problem or challenge that requires students to show what they know and can do. This task will address all four categories of the achievement chart.	• Addresses all four categories of the achievement chart; • Includes multiple overall curriculum expectations; • Provides flexibility in how students can demonstrate their learning (written, oral, construction).
Precise and Differentiated Instruction	• Teaching and learning experiences that respond to the individual learning needs of students; • Instruction is adjusted based on the observations from continuous assessment of student work; • Explicit descriptive feedback is provided to students, enabling them to set appropriate learning goals and improve their achievement.	• Continuous assessment to customize teaching; • Teachers give descriptive feedback verbally and in written form to students to support next level of performance; • Students use success criteria and descriptive feedback to set personal goals and learning pathways with or without teacher's and/or peer's assistance; • Instruction is personalized with consideration for learning and social factors (ethno-culture, class, gender, etc.).

CONTINUED ⟶

LEARNING SKILLS/MATHEMATICAL PROCESSES

- Are embedded within individual math strands;
- Students should be actively engaged in applying these process expectations throughout all math classes and courses;
- Processes are interconnected and interrelated, particularly Problem Solving & Communicating;
- Are fundamental to students' construction of their knowledge and skills related to mathematics; and
- Allow students to monitor and reflect on their learning (metacognition).
- Central to the actions of doing mathematics;
- Ways of acquiring and using the content, knowledge and skills of mathematics;
- Linked to three of the categories of the Achievement Chart-Thinking, Communication, and Application;
- Knowledge and Understanding, connects to the content of each math class and course; and
- Life-long learners of mathematics build new knowledge and skills in prior knowledge using mathematical processes.

COMPONENTS	K	1-3	4-6	7-10	11-12
Problem Solving	• Children begin to develop and apply problem-solving strategies, and persevere when solving problems and conducting mathematical investigations.	• Apply developing problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.	• Develop, select, and apply, problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.	• Develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.	• Develop, select, apply, compare and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding.
Reasoning and Proving	• Children apply developing reasoning skills to create and investigate possibilities.	• Apply developing reasoning skills to make and investigate conjectures.	• Develop and apply reasoning skills to make and investigate conjectures and construct and defend arguments.	• Develop and apply reasoning skills to make mathematical conjectures, assess conjectures and justify conclusions, and plan and construct organized mathematical arguments.	• Develop and apply reasoning skills to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments.
Reflecting	• Children demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.	• Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.	• Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.	• Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.	• Demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem.
Selecting Tools & Computational Strategies	• Children select and use a variety of concrete, visual, and electronic learning tools and appropriate strategies to investigate mathematical ideas and to solve problems.	• Select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.	• Select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.	• Select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.	• Select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems.
Connecting	• Children begin to make connections among mathematical concepts and notice examples of mathematics in their everyday life.	• Make connections among simple mathematical concepts and procedures, and relate mathematical ideas to situations drawn from everyday contexts.	• Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts.	• Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts.	• Make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts.
Representing	• Children create basic representations of simple mathematical ideas, make connections among them, and apply them to solve problems.	• Create basic representations of simple mathematical ideas, make connections among them, and apply them to solve problems.	• Create a variety of representations of mathematical ideas, make connections among them, and apply them to solve problems.	• Create a variety of representations of mathematical ideas, connect and compare them, and select and apply the appropriate representations to solve problems.	• Create a variety of representations of mathematical ideas, connect and compare them, and select and apply the appropriate representations to solve problems.
Communicating	• Children communicate mathematical thinking orally and visually, using everyday language, an emerging mathematical vocabulary, and a variety of representations.	• Communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.	• Communicate mathematical thinking orally, visually, and in writing, using everyday language, a basic mathematical vocabulary, and a variety of representations, and observing basic mathematical conventions.	• Communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.	• Communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

NUMERACY/MATHEMATICS BLOCK A THREE-PART FRAMEWORK

An information and technology-based society requires individuals who are able to think critically about complex issues, people who can "analyze and think logically about new situations, devise unspecified solution procedures, and communicate their solution clearly and convincingly to others" (Baroody, 1998, p.2-1).

To prepare students to function in such a society, teachers have a responsibility to promote in their classrooms the experience of problem-solving processes and the acquisition of problem-solving strategies, and to foster in students positive dispositions towards problem solving.

INQUIRY

PART ONE - GETTING STARTED / MINDS ON

Teacher Actions

- Activate students' prior knowledge;
- Mentally engage students in the problem-solving situation by posing a thought-provoking problem;
- Model tool and strategy selection as appropriate;
- Gather diagnostic and/or formative assessment data through observation and questioning;
- Discuss and clarify the task, including having students restate the problem in their own words and ask questions;
- Establish expectations; and
- Establish a positive mathematics classroom climate.

Student Actions

- Participate in discussions;
- Propose strategies;
- Question the teacher and their classmates; and
- Make connections to and reflect on prior learning.

PART TWO - WORKING ON IT / ACTION

Teacher Actions

- Facilitate student learning by:
  - √ providing hints and suggestions;
  - √ encouraging testing of ideas;
  - √ suggesting extensions of generalization;
  - √ asking probing questions;
  - √ answering students' questions to clarify mathematical misconceptions (but avoid providing a solution to the problem);
  - √ encourage students to represent their thinking;
- Observe and assess;
- Reconvene the whole group if significant questions arise;
- Encourage students to clarify ideas and to pose questions to other students (math talk); and
- Make connections with literacy and learning for life.

Student Actions

- Participate actively in whole group, small group, or independent settings;
- Explore and develop strategies and concepts;
- Select appropriate tools and strategies;
- Represent their thinking in a variety of ways;
- Develop and reflect upon alternative solutions;
- Engage in metacognition; and
- Communicate their understanding to their classmates and the teacher.

INQUIRY

PART THREE - A. REFLECT AND CONNECT / CONSOLIDATE AND DEBRIEF

Teacher Actions

Using Gallery Walk, Math Congress, Bansho to:

- Facilitate whole class discussion and sharing by:
  - √ bringing students back together to share and analyse solutions and address misunderstandings or confusions;
  - √ encouraging students to explain a variety of solution strategies;
  - √ asking students to defend their procedures and justify their answers;
  - √ engaging all class members.
- Connect strategies and solutions to similar types of problems in order to help students generalize concepts; and
- Ask clarifying and extending questions.

Student Actions

- Justify and explain their thinking and understanding with clarity and precision;
- Compare a variety of concrete, pictorial, and numerical representations;
- Listen and contribute to reflections on alternative approaches, different solutions, as well as extensions and connections;
- Reflect on their learning; and
- Retell, rephrase, and/or expand on mathematical ideas.

PART THREE - B. HIGHLIGHTS / SUMMARY

Teacher Actions

- Facilitate whole class discussion and reflection by:
  - √ probing students to summarize the discussion and emphasizing key points or concepts, i.e., "pulling out the math ",
- Record key mathematical concepts, vocabulary, algorithms, strategies in a list; and
- Make the learning from the lesson explicit.

Student Actions

- Articulate main points and ideas; and
- Express mathematical ideas and make connections.

PRACTICE & INQUIRY

PART THREE - C. PRACTICE

Teacher Actions

- Provide opportunities to practice
  - √ skills, problem solving, visualization, communication, metacognition;
- Plan the type and amount of practice based on the outcome of the lesson; and
- Differentiate the type and amount of practice based on student needs.

Student Actions

- Participate actively in whole group, small group, or independent settings; and
- Record solutions, thinking, representations, strategies.

	K-2	3-5	6-9	10-12
NUMERACY/ MATHEMATICS BLOCK	• Kindergarten:10-15 minutes of inquiry facilitated by the teacher • Grades1-2:30-45 minutes of inquiry and 10-15 minutes practice	• 45 minutes of inquiry and 10-15 minutes of practice	• 45-60 minutes of inquiry and 10-15 minutes of practice	• 60 minutes of inquiry- and 15-30 minutes of practice