Successful planning and teaching enables students to consistently and independently:

- Are central to instruction and are used to improve and deepen understanding;
- Are used to discuss mathematical relationships, concepts and ideas; and
- Are used to provide a model for complex concepts.

Provide opportunities for understanding, consolidation and practice;

- Are used by students to solve problems and apply skills; and
- Enable students to develop communication in mathematics through visual representation.

<table>
<thead>
<tr>
<th>COMPONENTS</th>
<th>K-2</th>
<th>3-5</th>
<th>6-9</th>
<th>10-12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculators</td>
<td>1 line display calculators (TI 08)</td>
<td>2 line display calculators (TI 10 and TI 15)</td>
<td>2 line display calculators (TI 15)</td>
<td>Graphing calculators (T10 and T15)</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>Students use manipulatives and technology as tools for learning</td>
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<td>Students use manipulatives and technology as tools for learning</td>
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<tr>
<td>Computers</td>
<td>Students explore and use maniplu- tives and technology as tools for learning</td>
<td>Computer programs and applications</td>
<td>Computer programs and applications (e.g., spreadsheets)</td>
<td>Dynamic geometry software</td>
</tr>
<tr>
<td>Interactive White Boards</td>
<td>Computer programs and applications</td>
<td>Dynamic geometry software</td>
<td>Dynamic geometry software</td>
<td>Dynamic geometry software</td>
</tr>
<tr>
<td>Student Response Systems</td>
<td>Dynamic statistical software</td>
<td>Internet</td>
<td>Internet</td>
<td>Internet</td>
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<tr>
<td>Document Cameras</td>
<td>e-Texts</td>
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<tr>
<td>Assistive Technology</td>
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<tr>
<td>e-Texts</td>
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</table>

Mathematics reveals patterns, order and relationships which help us to understand the world around us.

It is an increasingly varied discipline that deals with number, spatial sense, data, measurement, and observations requiring inference, deduction, modeling and communication. Alphanumeric is the application of mathematical concepts and procedures in a variety of contexts. It is interdisciplinary as it uses mathematics as a tool to explore problems and situations throughout the curriculum and every day life.

The Learning Destination for All Students in Mathematics

The Toronto District School Board identifies the teaching and learning of mathematics as a priority and is committed to providing support so that all learners achieve the highest academic levels.

(Taken from TDSB Mathematics Foundation Statement P004 CUR)

<table>
<thead>
<tr>
<th>The Effective Mathematics Classroom Environment</th>
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</thead>
<tbody>
<tr>
<td>• Both teachers and students demonstrate a positive attitude towards mathematics</td>
</tr>
<tr>
<td>• Is a safe, supportive and respectful classroom</td>
</tr>
<tr>
<td>• Includes engaging, challenging and authentic mathematics environments</td>
</tr>
<tr>
<td>• High expectations are held by teachers and students</td>
</tr>
<tr>
<td>• Provides collaborative learning opportunities</td>
</tr>
<tr>
<td>• Student mathematical thinking and learning is evident and posted</td>
</tr>
<tr>
<td>• Students generated work and co-constructed with the teacher is posted and clearly defined</td>
</tr>
<tr>
<td>• Students have easy access to and choice of learning tools, mathematics resources and technologies</td>
</tr>
<tr>
<td>• Includes activities and resources that are inclusive and reflect the needs of students with varying backgrounds, abilities, interests and learning styles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Professional Learning in Mathematics</th>
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</thead>
<tbody>
<tr>
<td>Teachers engage in collaborative professional learning, such as:</td>
</tr>
<tr>
<td>• Co-planning</td>
</tr>
<tr>
<td>• Co-teaching</td>
</tr>
<tr>
<td>• Lesson study</td>
</tr>
<tr>
<td>• Inquiry</td>
</tr>
<tr>
<td>• Webinars</td>
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<tr>
<td>• Modular learning</td>
</tr>
<tr>
<td>• Articulate clear understanding of mathematical concepts and ideas and give clear examples of use</td>
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<td>• U</td>
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<table>
<thead>
<tr>
<th>Characteristics of Mathematical Literate Students</th>
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<tbody>
<tr>
<td>• Can communicate their mathematical reasoning and can understand the mathematical reasoning of others</td>
</tr>
<tr>
<td>• Have a sense of numbers and are sufficiently efficient in their work that their thinking builds as they progress towards a solution</td>
</tr>
<tr>
<td>• To make sense of mathematics</td>
</tr>
<tr>
<td>• Make connections between concepts and see pattern throughout mathematics</td>
</tr>
<tr>
<td>• Are willing to persevere in order to understand and solve mathematical problems</td>
</tr>
<tr>
<td>• Are developing depth and flexibility in their thinking</td>
</tr>
</tbody>
</table>

(Adapted from Reporting in Mathematics 2010, Ann Davies)
COMPREHENSIVE MATHEMATICS PROGRAM

- Is focused on having students make sense of mathematics;
- Is based on problem solving and investigation of important mathematical concepts;
- Begins with the students’ understanding and knowledge of the topic;
- Includes students as active, rather than passive participants in their learning;
- Allows students to engage in a variety of problem solving experiences to develop conceptual and procedural understanding and skills through exploration, investigation, direct instruction and practice;
- Has students communicate and investigate their thinking through ongoing discussion;
- Includes all students, whether in the choice of problems or in the communicating of mathematical ideas;
- Incorporates ongoing assessment of student understanding to guide future instruction;
- Allows students to develop their ability to ask questions and to plan investigations to answer these questions;
- Enables students to spend most of their time working in partners or small groups to explore and learn concepts, complete tasks, represent, justify, and consolidate their thinking;
- Engages students in investigations or questions prompted by literature or a topic in other subject areas;
- Includes opportunities for students to apply math knowledge and skills in other subject areas;
- Integrates parents/caregivers as partners in supporting students disposition and achievement;
- Includes students as active, rather than passive participants in their learning;
- Begins with the students’ understanding and knowledge of the topic;
- Is based on problem solving and investigation of important mathematical concepts;
- Is focused on having students make sense of mathematics;
- Allows students to develop their ability to ask questions and to plan investigations to answer these questions;
- Incorporates effective Literacy strategies;
- Incorporates ongoing assessment of student understanding to guide future instruction;
- Allows students to communicate and justify thinking and strategies;
- Are set within a rich context;
- Promote students thinking about the big ideas in mathematics through an inter-strand approach;
- Are used within a three part lesson framework;
- Uses optimal groupings to encourage collaborative discourse and critical thinking.

INSTRUCTIONAL STRATEGIES IN MATHEMATICS

- Are used within a three part lesson framework;
- Are based on a problem solving or investigation approach;
- Include a balance of instructional methods including play, exploration, investigation, direct instruction, and practice;
- Incorporate effective Literacy strategies;
- Promote students thinking about the big ideas in mathematics through an inter-strand approach;
- Are set within a rich context;
- Allow students to communicate and justify thinking and strategies;
- Allow for differentiated learning within flexible groupings.

Strategies for Thinking and Consolidation

**Gallery Walk**
- Students and teachers circulate to examine student recorded solutions to a lesson problem
- Students read solutions and give oral and written feedback
- Teachers assess the range of mathematical thinking in the different solutions
- Teachers use the information to determine the focus of consolidation and next steps for planning

**Math Congress**
- Two or three student solutions are used to conduct a whole class discussion that will develop every student’s mathematical thinking
- Teachers use student solutions to prompt them to reason about big math ideas
- Specific ideas and strategies are generalized and connections made to previous math discussions and learning
- Students defend and support their solutions and thinking
- Teachers use questioning to prompt all students to reason and generalize based on the lesson goals

**Banho**
- Means board writing in Japanese
- Math thinking is organized and recorded as it occurs in the lesson
- Mathematical expressions, diagrams, solutions and strategies are recorded for all to engage in
- Various solutions are organized and compared through teacher and student questioning
- Students deepen their mathematical thinking through comparing generalizing and summarizing

**Assessment for and As Learning**

- Data Collection Methods
  - K-2: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 3-5: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 6-9: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 10-12: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing

- Components
  - K-2: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 3-5: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 6-9: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing
  - 10-12: Learning Goals, Success Criteria, Descriptive feedback, Student interviews, Observational data, Student work, Student models using manipulatives, Exit Cards, Reflective Journals, Peer and self assessment items, Portfolio, Peer editing

- Descriptor
  - Teacher Moderation
  - Peer and Self Assessment
  - Portfolio
  - Peer editing
  - Descriptive feedback

- Look fors
  - Professional learning is focused on having students work and taking student(s) to the next level of learning;
  - Structural regular opportunities to examine student work collaboratively in the classroom setting;
  - Calibrate standards of expectations and practice among teachers within and across grades;
  - Develop school-wide beliefs, values and practices that support students’ well being and success; and
  - School staff can explain what is the difference that makes the difference for their students.

- Students have regular and structural opportunities to speak to each other about their progress and their work;
- Students can describe what success looks like based on set criteria; and
- Students have a clear understanding and can articulate why they are learning, what and how they are learning.

Adapted from: Communication in the Mathematics Classroom, Special Edition #13 Literacy and Numeracy Secretariat

Revised April 2013
ASSESSMENT FOR AND AS LEARNING

- ongoing assessment
- demonstrates what students already know
- informs instruction that is differentiated and personalized
- monitors students progress

COMPONENTS

<table>
<thead>
<tr>
<th>Student Work</th>
<th>Look For</th>
</tr>
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<tbody>
<tr>
<td>Through facilitated discussions with students and teacher moderation of student work, teachers assess student learning, plan and provide feedback, and adjust instruction.</td>
<td>Student work used to share strategies and solutions, consolidate learning, prompt mathematical communication; Student work samples guide professional learning and decision making.</td>
</tr>
</tbody>
</table>

Learning Goals

- What students should be able to do by the end of a period of instruction (e.g. a lesson, series of lessons, or subtask).
- Based on expectations for performance as stated in all four areas of the achievement chart.

Success Criteria

- A description of successful attainment of the learning goals that shows what success "looks like".
- Success criteria are used to guide descriptive feedback that informs the next level of teaching and learning.

Rich Assessment Task

- Authentic activity, exercise, problem or challenge that requires students to show what they know and can do. This task will address all four categories of the achievement chart.

Precise and Differentiated Instruction

- Teaching and learning experiences that respond to the individual learning needs of students;
- Instruction is adjusted based on the observations from continuous assessment of student work;
- Explicit descriptive feedback is provided to students, enabling them to set appropriate learning goals and improve their achievement.

CONTINUED
A Three-Part Framework

**Part One - Getting Started / Minds On**

**Teacher Actions**
- Activate students’ prior knowledge;
- Mentally engage students in the problem-solving situation by posing a thought-provoking problem;
- Model tool and strategy selection as appropriate;
- Gather diagnostic and/or formative assessment data through observation and questioning;
- Discuss and clarify the task, including having students restate the problem in their own words and ask questions;
- Establish expectations; and
- Establish a positive mathematics classroom climate.

**Student Actions**
- Participate in discussions;
- Propose strategies;
- Question the teacher and their classmates; and
- Make connections to and reflect on prior learning.

**Part Two - Working on It / Action**

**Teacher Actions**
- Facilitate student learning by:
  - providing hints and suggestions;
  - encouraging testing of ideas;
  - suggesting extensions of generalization;
  - asking probing questions;
  - engaging students’ questions to clarify mathematical misconceptions (but avoid providing a solution to the problem);
  - having students present their thinking;
  - Observe and assess;
  - Reconvene the whole group if significant questions arise;
  - Encourage students to clarify ideas and to pose questions to other students (math talk); and
  - Make connections with literacy and learning for life.

**Student Actions**
- Participate actively in whole group, small group, or independent settings;
- Explore and develop strategies and concepts;
- Select appropriate tools and strategies;
- Represent their thinking in a variety of ways;
- Develop and reflect upon alternative solutions;
- Engage in metacognition; and
- Communicate their understanding to their classmates and the teacher.

**Part Three - Reflect and Connect / Consolidate and Debrief**

**Teacher Actions**
- Facilitate whole class discussion and sharing by:
  - bringing students back together to share and analyse solutions and address misunderstandings or confusions;
  - encouraging students to explain a variety of solution strategies;
  - asking students to defend their procedures and justify their answers;
  - engaging all class members;
  - Connect strategies and solutions to similar types of problems in order to help students generalize concepts; and
  - Ask clarifying and extending questions.

**Student Actions**
- Justify and explain their thinking and understanding with clarity and precision;
- Compare a variety of concrete, pictorial, and numerical representations;
- Listen and contribute to reflections on alternative approaches, different solutions, as well as extensions and connections;
- Reflect on their learning; and
- Rethink, rephrase, and/or expand on mathematical ideas.

**Part Three - Highlights / Summary**

**Teacher Actions**
- Facilitate whole class discussion and reflection by:
  - probing students to summarize the discussion and emphasizing key points or concepts, i.e., “pulling out the math”;
  - Record key mathematical concepts, vocabulary, algorithms, strategies in a list; and
  - Make the learning from the lesson explicit.

**Student Actions**
- Articulate main points and ideas; and
- Express mathematical ideas and make connections.

**Part Three - Practice**

**Teacher Actions**
- Provide opportunities to practice
  - skills, problem solving, visualization, communication, metacognition;
  - Plan the type and amount of practice based on the outcome of the lesson; and
  - Differentiate the type and amount of practice based on student needs.

**Student Actions**
- Participate actively in whole group, small group, or independent settings; and
- Record solutions, thinking, representations, strategies.

**K-2**
- Kindergarten: 10-15 minutes of inquiry facilitated by the teacher
- Grade 1: 20-30 minutes of inquiry and 10-15 minutes of practice

**3-5**
- 45 minutes of inquiry and 10-15 minutes of practice

**6-9**
- 45-60 minutes of inquiry and 10-15 minutes of practice

**10-12**
- 60 minutes of inquiry and 15-30 minutes of practice

Revised April 2013