

$2+2=4$

+

x

Junior Break Out Room

42:9

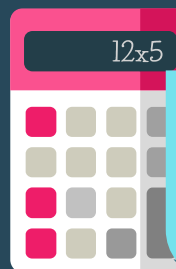
$\sqrt[n]{x}$

A Focus on Multiplication

%

$x/2y$

-



a

Team Members



Jamile Lewis

Middle Years Student
Success Counsellor



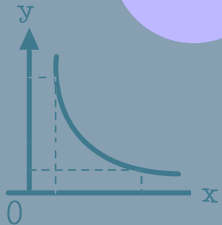
Uloma Onyido

Middle Years Student
Success Counsellor



Marcus Hagley

K - 12 Math Coach



+

x

%



What We Will Learn

- + Strategies to help solve multiplication problems
- + Strategies for deriving multiplication facts
- + Common areas of struggle for students when solving multiplication problems.



Minds On

There were 24 kids in the class.

Each one paid \$15 for the field trip.

How much money was collected?

Reflect on the first strategy that came to mind.



$x/2y$

Minds On Solutions

+

x

Decompose by Place Value

$$\begin{aligned} 24 \times 15 &= 24 \times 10 + 24 \times 5 \\ &= 240 + 120 \\ &= 360 \end{aligned}$$



$x/2y$

Minds On Solutions

+

x

halving & doubling

$$24 \times 15 = 12 \times 30 = 6 \times 60 = 360$$



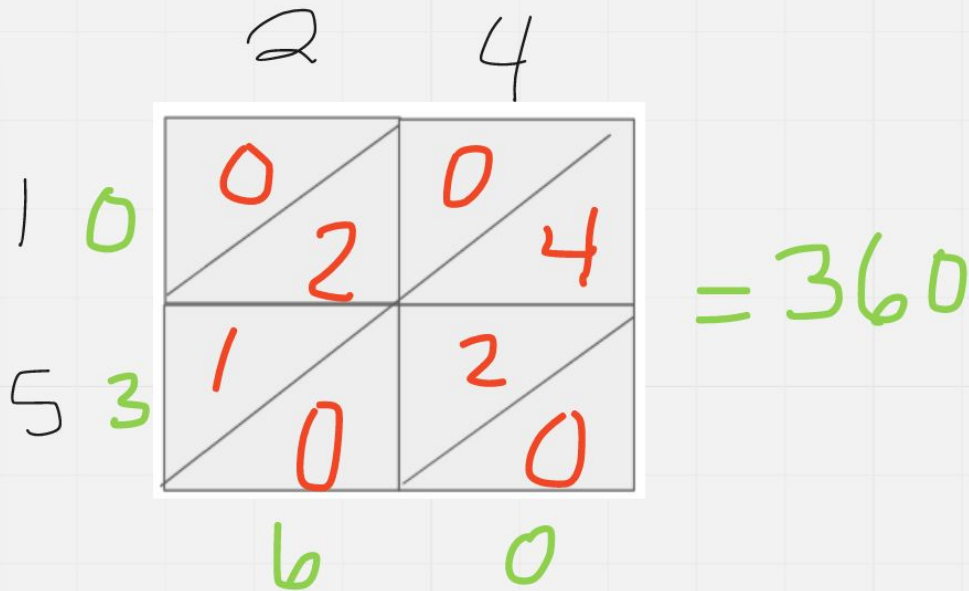
$x/2y$

Minds On Solutions

+

x

lattice method



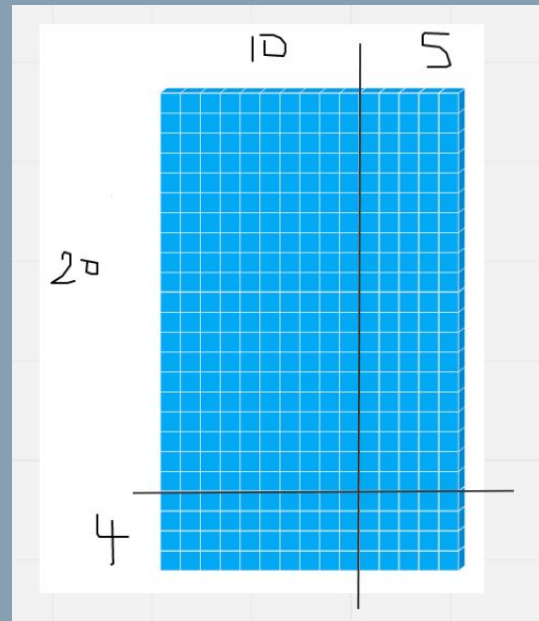
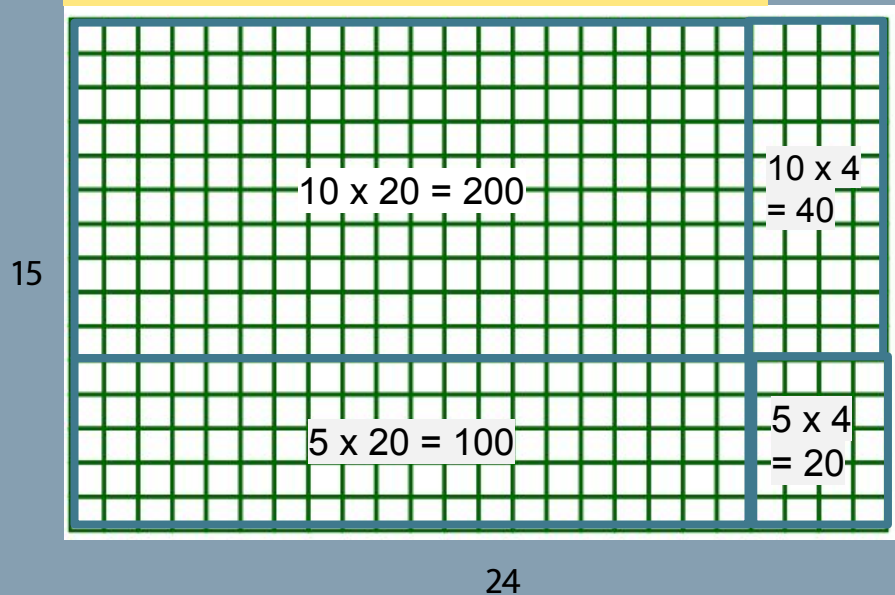
$x/2y$

Minds On Solutions

+

x

area model

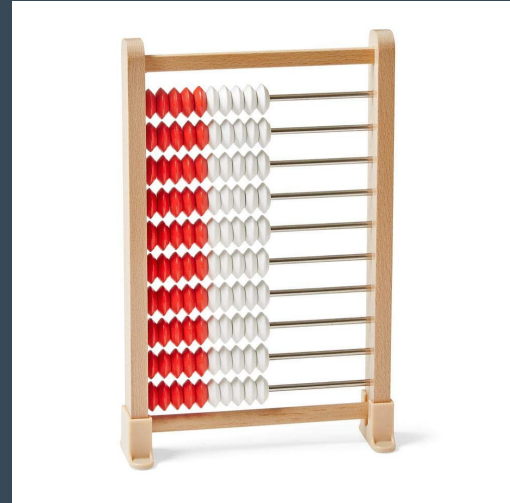


What is my child expected to know/do?

Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Math Facts							
B2.2 recall and demonstrate addition facts for numbers up to 10, and related subtraction facts	B2.2 recall and demonstrate addition facts for numbers up to 20, and related subtraction facts	B2.2 recall and demonstrate multiplication facts of 2, 5, and 10, and related division facts	B2.2 recall and demonstrate multiplication facts for 1×1 to 10×10 , and related division facts	B2.2 recall and demonstrate multiplication facts from 0×0 to 12×12 , and related division facts	B2.2 understand the divisibility rules and use them to determine whether numbers are divisible by 2, 3, 4, 5, 6, 8, 9, and 10	B2.2 understand and recall commonly used percents, fractions, and decimal equivalents	B2.2 understand and recall commonly used square numbers and their square roots
Multiplication and Division							
		B2.6 represent multiplication of numbers up to 10×10 and division up to $100 \div 10$, using a variety of tools and drawings, including arrays			B2.6 represent composite numbers as a product of their prime factors, including through the use of factor trees	B2.6 determine the greatest common factor for a variety of whole numbers up to 144 and the lowest common multiple for two and three whole numbers	
B2.5 represent and solve equal-group problems where the total number of items is no more than 10, including problems in which each group is a half, using tools and drawings	B2.5 represent multiplication as repeated equal groups, including groups of one half and one fourth, and solve related problems, using various tools and drawings	B2.7 represent and solve problems involving multiplication and division, including problems that involve groups of one half, one fourth, and one third, using tools and drawings	B2.5 represent and solve problems involving the multiplication of two- or three-digit whole numbers by one-digit whole numbers and by 10, 100, and 1000, using appropriate tools, including arrays	B2.6 represent and solve problems involving the multiplication of two-digit whole numbers by two-digit whole numbers using the area model and using algorithms, and make connections between the two methods	B2.7 represent and solve problems involving the multiplication of three-digit whole numbers by decimal tenths, using algorithms	B2.7 evaluate and express repeated multiplication of whole numbers using exponential notation, in various contexts	

Developing Math Fact Fluency

- **Helping our children to develop concepts and strategies must come first before drill, practice, and memorization of basic math facts.**
- **Our children need multiple experiences developing strategies for finding products of basic math facts because they become more fluent and approach automaticity with their math facts.**



Developing Math Fact Fluency: Strategies

Commutative Property (Partner Facts)

- Used to derive turn-around facts

Example:

$$5 \times 6 = 6 \times 5$$



Developing Math Fact Fluency: Strategies

The Identity Property of Multiplication

- The product of any number and 1 is that number

Example:

$$6 \times 1 = 6$$

$$1 \times 6 = 6$$



Developing Math Fact Fluency: Strategies

Doubling

- Facts with an even number as one of the factors

Example:

$$7 \times 2 = 7 + 7 = 14$$

$$7 \times 4 = (7 \times 2) \times 2$$

$$14 \times 2 = 14 + 14 = 28$$

Example:

$$7 \times 8 = (7 \times 4) \times 2$$

$$28 \times 2 = 28 + 28 = 56$$

Developing Math Fact Fluency: Strategies

Skip Counting

- Facts in which a skip counting pattern of one of the factors is known

Example:

$$7 \times 5 = 35$$

5, 10, 15, 20, 25, 30, 35

$$5 \times 10 = 50$$

10, 20, 30, 40, 50.

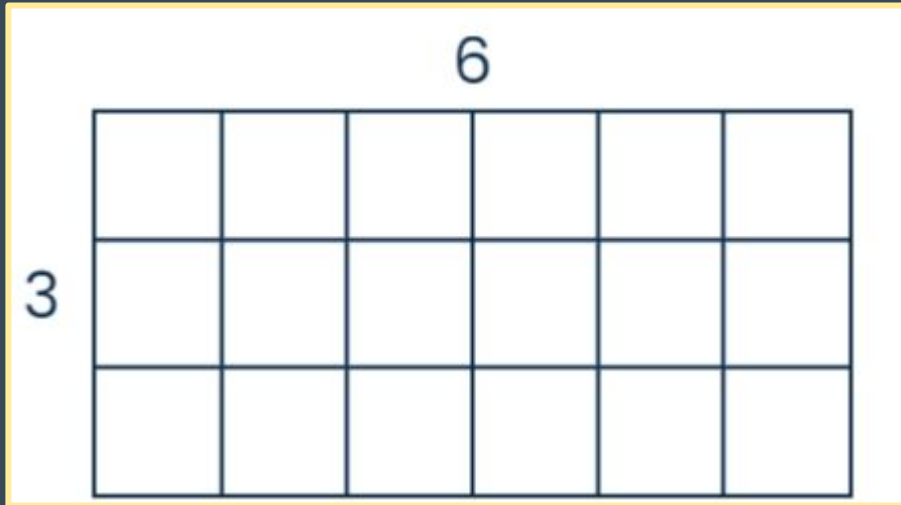


Developing Math Fact Fluency: Strategies

Area Model

- Making a sketch using the understanding of the relationship between the dimensions and area of a rectangle with the ability to see and count the squares.

Example: 6×3



Developing Math Fact Fluency: Strategies

Open Area Model

- Making a sketch using the understanding of the relationship between the dimensions and area of a rectangle and the distributive property.

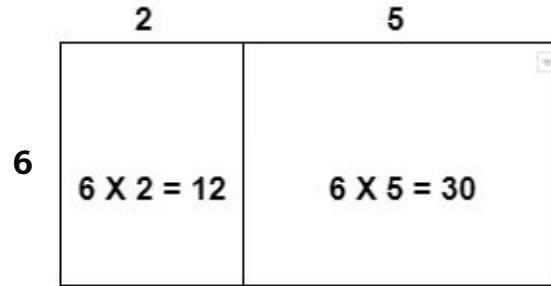
Example:

$$6 \times (2 + 5)$$

$$6 \times 2 = 12$$

$$6 \times 5 = 30$$

$$12 + 30 = 42$$



$$\begin{array}{r} 12 \\ + 30 \\ \hline 42 \\ \hline \end{array}$$

Developing Math Fact Fluency: Strategies

Distributive Property and anchor facts

- Any fact by using a known or anchor fact (friendly number) and the distributive property.

Example: 6×9



$$\begin{aligned}6 \times 5 &= 30 \\6 \times 4 &= 24 \\30 + 24 &= 54\end{aligned}$$

6×9



$$\begin{aligned}6 \times 9 &= \\(6 \times 5) &+ (6 \times 2) + (6 \times 2) \\30 &+ 12 + 12 \\&= 54\end{aligned}$$

Developing Math Fact Fluency: Strategies

Inverse relationship between multiplication and division

- Division facts

Example:

$$54 \div 6 = ?$$

$$6 \times ? = 54$$

From Fluency to Automaticity

Applying the identity property of multiplication, the commutative property, and multiplication by 0 reduces the number of facts to learn from 121 to a little less than 50.

- Commutative Property (Fact Partners): 3×4 and 4×3
- Multiplication by 0
- Identity Property of Multiplication: $1 \times a = a$



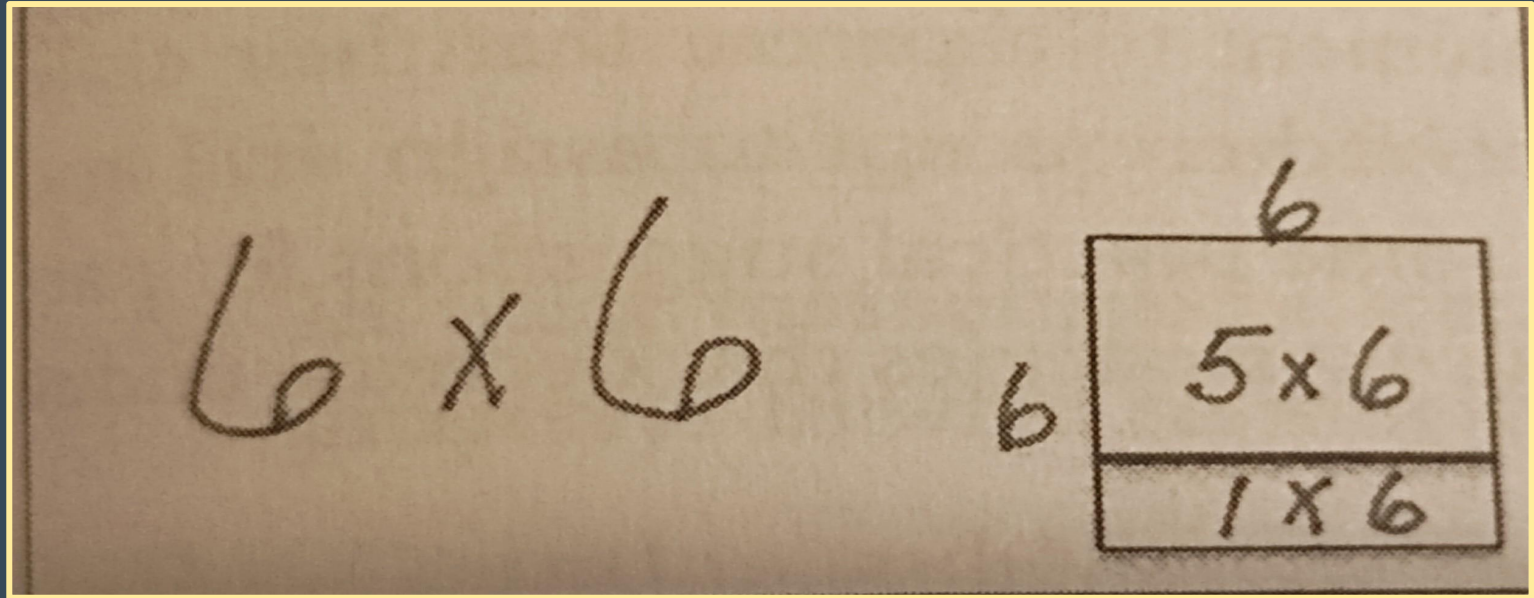
From Fluency to Automaticity

- Multiplication math fact table
- Shaded sections shows the facts understood through the commutative property

X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

From Fluency to Automaticity

- Targeted Fact Practice using flash cards
- Have your child create a visual model clues on the flash cards to develop strategy for deriving facts



General Facts Practice

- General fact practice using a variety of online games and activities
- Effortless fact practice: For example, post facts your child needs to become automatic with around the house in places where they see often.

“Development of fluency and automaticity with math facts takes a multifaceted approach that incorporate development of fluency using strategies that develop understanding and then the implementation of targeted and general practice to achieve the goal of automaticity.” Source: A Focus on Multiplication & Division by Elizabeth Hulbert et. al.



Conceptual Understanding is Important

Usha has memorized her times tables up to 5. While doing her homework, she encounters the following question:

There are 6 groups of students with 6 in each group.

How many students are there altogether?

Usha feels stuck. She does not remember her “6 facts” and can’t think of what to do next.

Peter Liljedahl has said that “problem solving is what we do when we don’t know what to do.”

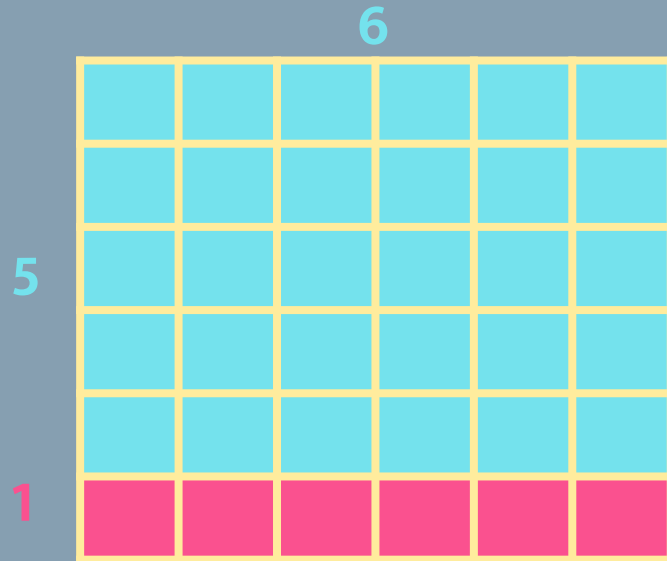
We would like to see Usha use her understanding of 5 facts to help her solve this problem.

She might say...

“I know that 5 groups of 6 is 30, so one more group of 6 will be 36.”

Why some students struggle...

They may not see the connection between facts. Like Usha in our previous example, they may not see that 5×6 and 6×6 are related.



x

+

Why some students struggle...

x

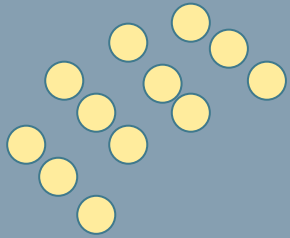
They may not have consolidated earlier skills related to addition, skip counting, place value, knowledge of basic facts. Targeted instruction is required to help them close these gaps.

Counting by Ones

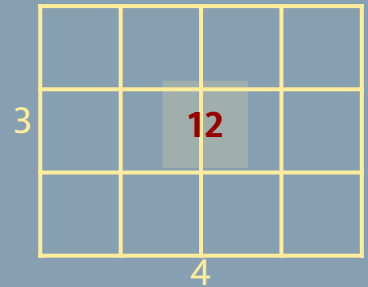
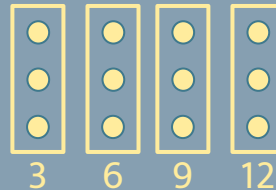
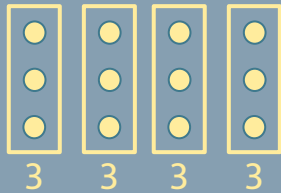
Repeated Addition

Skip Counting

Multiplication



$$3 + 3 + 3 + 3 = 12$$



x + Why some students struggle... x

% They may not understand the commutative, associative, or distributive properties of multiplication. y

Commutative Property

The order of the numbers (factors) in a multiplication sentence does not change the result (product).

$$4 \times 3 = 12$$

$$3 \times 4 = 12$$

Associative Property

When you multiply, you can group the numbers in any combination.

$$4 \times (5 \times 3) = (4 \times 5) \times 3$$

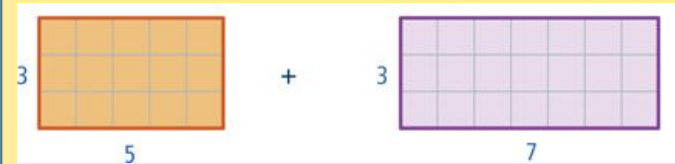
$$4 \times 15 = 20 \times 3$$

$$60 = 60$$

$$4 \times 15 = 2 \times 30$$

Half and Double strategy

Distributive Property



$$3 \times 12 = 3(5+7) = (3 \times 5) + (3 \times 7)$$

Partial Products strategy


x + Why some students struggle... x

More emphasis may have been placed on memorization over strategy development and a conceptual understanding of multiplication


Symbolic

Name: _____ Score: _____ out of 44
Time: _____ minutes

Multiplication: 0 - 2

a. $\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 3 \\ \hline \end{array}$ 


b. $\begin{array}{r} 0 \\ \times 1 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 5 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 10 \\ \hline \end{array}$

c. $\begin{array}{r} 1 \\ \times 1 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 10 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 1 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 10 \\ \hline \end{array}$ 

d. $\begin{array}{r} 0 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 5 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 0 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 2 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 0 \\ \hline \end{array}$


e. $\begin{array}{r} 0 \\ \times 0 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 7 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 9 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 8 \\ \hline \end{array}$

f. $\begin{array}{r} 0 \\ \times 4 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 3 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 5 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 10 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 0 \\ \hline \end{array}$

g. $\begin{array}{r} 1 \\ \times 6 \\ \hline \end{array}$  $\begin{array}{r} 0 \\ \times 7 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 7 \\ \hline \end{array}$ $\begin{array}{r} 2 \\ \times 10 \\ \hline \end{array}$ $\begin{array}{r} 0 \\ \times 6 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ \times 4 \\ \hline \end{array}$

Super Teacher Worksheets - www.superteacherworksheets.com

Verbal
Physical
Visual
Contextual



Write a problem that could be represented by the counters shown here



x

+

x

%

y

Resources

[Number Talks at Home - TDSB Mathematics for Families and Caregivers Website](#)

[TDSB Virtual Math Toolpage](#)

[Mathplayground - Multiplication and Division games](#)


[Sheppard Software - Multiplication Games](#)



x + % Thank You! x y

Questions?

If you are able, please ask your question in the chat

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